FORMULATION OF DESIGN TABLES AND CHARTS FOR DESIGN OF REINFORCED CONCRETE SLABS

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SUBMITTED TO

THE DEPARTMENT OF CIVIL ENGINEERING

FACULTY OF ENGINEERING

NNAMDI AZIKIWE UNIVERSITY AWKA.

IN PARTIAL FUFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF BACHELOR OF ENGINEERING (B.ENG) DEGREE IN CIVIL ENGINEERING

MAY, 2023

CERTIFICATION

This is to certify that this project topic titled "Formulation of Design Tables and Charts for Design of Reinforced Concrete Slabs" was undertaken by Ubah Justine Chukunonso with registration number (NAU/2017224028) in the Department of Civil Engineering, Nnamdi Azikiwe University, Awka, Anambra State.

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APPROVAL PAGE

This research work "Formulation of Design Tables and Charts for Design of Reinforced Concrete" has been assessed and approved by department of civil engineering Nnamdi Azikiwe University.

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DEDICATION

This work is dedicated to Almighty God for the gift of life and also for guiding me through school.

ACKNOWLEDGEMENT

Special thanks go to Almighty God for giving me the strength to complete this work and also for His guidance and protection throughout my stay in Nnamdi Azikiwe University.

Also I will like to express my profound gratitude to my parents; Mr and Mrs Godwin Ubah for their moral support, constant prayers throughout my stay in school, special thanks goes also to my siblings Ifeoma, Kingsley, and Praises for their encouragement during trying times of my academic pursuit.

I want to thank in a very special way, my project supervisor in the person of Engr. Dr. V.O. Okonkwo for his time and guidance in the accomplishment of this project. May the lord enlarge your coast and also protect your family.

Special thanks go to the Head of Department of Civil Engineering in the person of Engr. Prof. C. A. Ezeagu. He has been more like a father to us all and I appreciate him so much.

I will also like to extend my heartfelt appreciation to all the staff (academic and non-academic) of the Department of Civil Engineering for their invaluable tutorship and professional guidance.

Finally, I will like to appreciate everyone who has in one way or the other contributed to making me a better person, may the Lord Almighty reward you all greatly

ABSTRACT

The study was undertaken to formulate design tables and charts for design of reinforced concrete slabs. The design tables and charts were developed based on mathematical derivation done in accordance with BS 8110. Several design charts were developed with respect to the ratio of effective depth to the overall depth of the slab cross section. The developed design charts and tables were restricted to design of singly reinforced concrete slab at both ultimate and serviceability limit state. Several design examples were solved using the developed tables and charts. Accuracy of the developed design tables and charts were assessed using BS 8110 design code of practice. It was deduced that the developed design tables and charts was reliable as reasonable agreement exist between the design output obtained using the developed design tables and charts and BS 8110 design solution.

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LIST OF SYMBOL & ABBREVIATION

RCS – Reinforced Concrete Slab

LL – Live load

DL- Dead load

SLS—Serviceability Limit State

ULS- Ultimate Limit State

BS – British Standard

LA – Lever arm

D – Overall depth of Slab

d = Effective depth of slab

BM – Bending Moment

Asreq= Area of steel

Aprov= Area of steel provided

SF- Shear Force

CHAPTER ONE

INTRODUCTION

1.1 Background of Study

Reinforced concrete is a combination of two dissimilar but complementary materials namely concrete and steel (Oyenuga, 2011). Concrete have considerable crushing strength, it is durable, have good fire resistance but offers little or no strength in tension but fair in shear (Oyenuga, 2011). On the other hand, steel have good tensile properties, poor resistance to fire (due to rapid loss of strength under high temperature) and is very good in both shear and compression. Thus a combination of these two materials results in good tensile and compressive strength, durability and good resistance to fire and shear (Oyenuga, 2011).

Reinforced concrete slab are plate elements which forms the floor of a structure and are more often subjected to bending (tensile or compressive) but in rare cases (such as bridge deck) subjected to shear (Oyenuga, 2011). Slabs are horizontal members but they can serve as vertical members such walls to infill panels, side walls to drain and sewer appurtenances (Oyenuga, 2011). The provision of adequate reinforcement, slab thickness and proper detailing to satisfy both ultimate and serviceability limit state forms the bases of the design of reinforced concrete slab (Oyenuga, 2011).

Reinforced concrete was invented during the second half of the 19^{th} century (Michel, Garibel and Curbach, 2018). Besides the need to substitute wood for gardening and recreational use, the main driver was the need for an economic and fire proof building materials (Michel, Garibel and Curbach, 2018). The development of modern cement and steel during the first half of the 19^{th} century made this invention possible (Michel, Garibel and Curbach, 2018).During the 1850s, two French horticulturist, Lambas and Monica started very empirically to build planter and other appliances with cement reinforced with iron or steel mesh. After the early works of these pioneers, this invention was widely developed in many countries which ultimately gained credence in Nigeria and other African countries.

Design of reinforced concrete slab as structural part of any civil engineering structure (bridge, building, culvert) is a complex problem which entails selection of manuals and standards for design, load estimation, analysis to generate the internal stresses (maximum bending moment and shear force) and design of slab section so as to determine the area of reinforcement. This process requires substantial amount of time and energy which may results to analysis and design flaws. It is therefore paramount that the design of reinforced concrete slab should be simplified for greater accuracy and efficiency. Formulation of design tables and charts for design of the reinforced concrete slabs becomes a viable alternative. In addition, charts for design of reinforced concrete slab are not explicitly specified in relevant code of practice (BS: 8110, 1985 and BS: 8110, 1997). Also, charts for design of reinforced concrete slab contained in most code of practice are not updated for new factor of safety for materials.

Sequel to the aforementioned, this study will sought out ways to minimize time and effort expended during analysis and design of reinforced concrete slab design with varying cross section by developing design tables and charts relevant to design of reinforced concrete slab.

1.2 Statement of Problem

Design of reinforced concrete slab entails selection of design manuals and standards, generation of internal stresses (maximum moment and shear force), design and use of design tables and charts to determine the percentage reinforcement and other design requirements (Oyenuga, 2011). Substantial amount of time and energy is expended by structural analysis and design engineers in the design of reinforced concrete slab with varying cross section (span and thickness). This is due to knowledge gap on means to ensure simplicity, accuracy and efficiency in the design of reinforced concrete slabs. The development of design tables and charts for specification of area of reinforcement at ultimate limit state of design for reinforced concrete slabs with varying cross section becomes an essentially viable alternative.

Moreover, the new factor of safety for materials specified by relevant code of practice are not updated in recent design charts used for design of reinforced concrete slab. In other to optimize reinforced concrete slab design, and ensure efficiency in design of reinforced concrete slab, this study will therefore develop a simplified design tables and charts for specifying the area of reinforcement at ultimate limit state of design of reinforced concrete slabs with varying cross section.

1.3 Aim and Objectives of Study

The aim of the study is the formulation of design tables and charts for the design of reinforced concrete slabs while the objectives are:

1. Select and study the design code of practice relevant to the design of reinforce concrete slabs.
2. Study the reinforced concrete design textbooks and manual for the method of design of reinforced concrete slab.
3. Formulation of design steps for the design of reinforced concrete slab.
4. Develop table and charts for the implementation of the designed slab.
5. Demonstrate the use of the new graph to assess their usefulness.

1.4 Scope of Study

The study is essentially centered at formulation of design tables and charts for design of reinforced concrete slabs with varying cross sections. Design tables and charts will be developed in accordance with the procedure stated in relevant code of practice (BS: 8110, 1997). Mathematical equation valuable to the formulation of the design charts and tables will be derived. Design tables and charts for specifying the area of reinforcement at the tension zone for reinforced concrete slabs with varying cross section will be developed. The design tables and charts for specification of percentage reinforcement for the compression zone will not be given due consideration as compression reinforcement is seldom in slab. The processes to be employed for the formulation of design tables and charts for the reinforced concrete slabs will be presented. Two slabs with varying cross section (thickness and span) usually of standard sizes will be designed and their respective area of reinforcement will be provided using the developed design charts and tables for reinforced concrete slabs with varying cross section. Results obtained from design of reinforced concrete slab using the developed design charts and tables will be compared with BS 8110 design code of practice in order to check for accuracy of the developed design tables and charts.

1.5 Significance of Study

The findings obtained from the study on formulation of design tables and charts for design of reinforced concrete slabs will however be significant in the following ways:

1. Foster efficiency in the design of reinforced concrete slab.
2. Simplify analysis and design of reinforced concrete slabs.
3. Ensure regular update of material factor of safety for design of reinforced concrete slabs.
4. Valuable as a body of knowledge for undergraduate and design professionals in the construction industry.

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Overview

Reinforced concrete slab are plate elements which forms the floor of a structure and are more often subjected to bending (tensile or compressive) but in rare cases (such as bridge deck) subjected to shear (Oyenuga, 2011). Slabs are horizontal members but they can serve as vertical members such walls to infill panels, side walls to drain and sewer appurtenances (Oyenuga, 2011). Slabs can be solid or ribbed, and can span between beams, in either one or two directions,

or be supported directly by columns as a flat slab (Reynord and Steedman, 2008). Slab elements occur also as decking in bridges and other forms of platform structures, and as walling in rectangular tanks, silos and other forms of retaining structures (Reynord and Steedman, 2008).According to (Oyenuga, 2011), there are various types of slab and slab types are influenced by span, use of space which may determine the span, load to be borne by the slab and architectural aesthetics. (Oyenuga, 2011) classified slabs types as solid slabs (cantilever, simply supported and two ways), ribbed floor slab, flat slab and waffle slabs. According to (Sabah, 2017), slabs are divided into suspended slabs, suspended slabs may be divided into two groups: slabs supported on edges of beams and walls and also slabs supported directly on columns without beams and known as flat slabs. Supported slabs may be one-way slabs (slabs supported on two sides and with main reinforcement in one direction only) and two-way slabs (slabs supported on four sides and reinforced in two directions).

This section will review relevant literatures on reinforced concrete slab and principles for formulation of design tables and charts for reinforced concrete slabs.

2.2 History Overview of Reinforced Concrete

Many researchers believe that the first use of a truly cementitious binding agent (as opposed to the ordinary lime commonly used in ancient mortars) occurred in southern Italy around second century BC. Volcanic ash (called pozzuolana, found near Pozzouli, by the Bay of Naples) was a key ingredient in the Roman cement used during the days of the Roman empire. Roman concrete bears little resemblance to modern Portland cement concrete. It was never put into a mould or formwork in a plastic state and made to harden, as is being done today. Instead, Roman concrete was constructed in layers by packing mortar by hand in and around stones of various sizes. The Pantheon, constructed in AD 126, is one of the structural marvels of all times (Shaeffer 1992).

During the Middle Ages, the use of concrete declined, although isolated instances of its use have been documented and some examples have survived. Concrete was more extensively used again during the Renaissance (14th–17th centuries) in structures like bridge piers. Pozzolanic materials were added to the lime, as done by the Romans, to increase its hydraulic properties (Reed, et al. 2008). In the eighteenth century, with the advent of new technical innovations, a greater interest was shown in concrete. In 1756, John Smeaton, a British Engineer, rediscovered hydraulic cement through repeated testing of mortar in both fresh and salt water. Smeaton's work was followed by Joseph Aspdin, a bricklayer and mason in Leeds, England, who, in 1824, patented the fi rst 'Portland' cement, so named since it resembled the stone quarried on the Isle of Portland off the British coast (Reed, et al. 2008). Aspdin was the first to use high temperatures to heat alumina and silica materials, so that cement was formed. It is interesting to note that cement is still made in this way. I.K. Brunel was the fi rst to use Portland cement in an engineering application in 1828; it was used to fill a breach in the Thames Tunnel. During 1959–67, Portland cement was used in the construction of the London sewer system.

The small rowboats built by Jean-Louis Lambot in the early 1850s are cited as the first successful use of reinforcements in concrete. During 1850–1880, a French builder, Francois Coignet, built several large houses of concrete in England and France (Reed, et al. 2008). Joseph Monier of France, who is considered to be the fi rst builder of RC, built RC reservoirs in 1872. In 1861, Monier published a small book, Das System Monier, in which he presented the applications of RC. During 1871–75, William E. Ward built the first landmark building in

RC in Port Chester, NY, USA. In 1892, François Hennebique of France patented a system of steel-reinforced beams, slabs, and columns, which was used in the construction of various structures built in England between 1897 and 1919. In Hennebique's system, steel reinforcement was placed correctly in the tension zone of the concrete; this was backed by a theoretical understanding of the tensile and compressive forces, which was developed by Cottançin in France in 1892 (Reed, et al. 2008).

2.3 Reinforced Concrete Design Process

There are two principal stages in the calculations required to design a reinforced concrete structure. In the first stage, calculations are made to determine the effect on the structure of loads and imposed deformations in terms of applied moments and forces. In the second stage, calculations are made to determine the capacity of the structure to withstand such effects in terms of resistance moments and forces.

Factors of safety are introduced in order to allow for the uncertainties associated with the assumptions made and the values used at each stage. For many years, unfactored loads were used in the first stage and total factors of safety were incorporated in the material stresses used in the second stage. The stresses were intended to ensure both adequate safety and satisfactory performance in service. This simple approach was eventually replaced by a more refined method, in which specific design criteria are set and partial factors of safety are incorporated at each stage of the design process. In modern Codes of Practice, a limit-state design concept is used. Ultimate (ULS) and serviceability (SLS) limit-states are considered, as well as durability and, in the case of buildings, fire-resistance. Partial safety factors are incorporated in both loads and material strengths, to ensure that the probability of failure (i.e. not satisfying a design requirement) is acceptably low.

Members are first designed to satisfy the most critical limit state, and then checked to ensure that the other limit-states are not reached. For most members, the critical condition to be considered is the ULS, on which the required resistances of the member in bending, shear and torsion are based. The requirements of the various SLSs, such as deflection and cracking, are considered later. However, since the selection of an adequate span to effective depth ratio to prevent excessive deflection, and the choice of a suitable bar spacing to avoid excessive cracking, can also be affected by the reinforcement stress, the design process is generally interactive. Nevertheless, it is normal to start with the requirements of the ULS.

2.4 Reinforced Concrete Design Criteria and Safety Consideration

A limit-state design concept is used in British and European Codes of Practice. Ultimate (ULS) and serviceability (SLS) limit states need to be considered as well as durability and, in the case of buildings, fire-resistance. Partial safety factors are incorporated into loads (including imposed deformations) and material strengths to ensure that the probability of failure (not satisfying a design requirement) is acceptably low.

In BS 8110 at the ULS, a structure should be stable under all combinations of dead, imposed and wind load. It should also be robust enough to withstand the effects of accidental loads, due to an unforeseen event such as a collision or explosion, without disproportionate collapse. At the SLS, the effects in normal use of deflection, cracking and vibration should not cause the structure to deteriorate or become unserviceable. A deflection limit of span/250 applies for the total sag of a beam or slab relative to the level of the supports. A further limit, the lesser of span/500 or 20 mm, applies for the deflection that occurs after the application of finishes, cladding and partitions so as to avoid damage to these elements. A limit of 0.3 mm generally applies for the width of a crack at any point on the concrete surface.

In BS 5400, an additional partial safety factor is introduced. This is applied to the load effects and takes account of the method of structural analysis that is used. Also there are more load types and combinations to be considered. At the SLS, there are no specified deflection limits but the cracking limits are more critical. Crack width limits of 0.25, 0.15 or 0.1 mm apply according to surface exposure conditions. Compressive stress limits are also included but in many cases these do not need to be checked. Fatigue considerations require limitations on the reinforcement stress range for unwelded bars and more fundamental analysis if welding is involved. Footbridges are to be analyzed to ensure that either the fundamental natural frequency of vibration or the maximum vertical acceleration meets specified requirements.

In BS 8007, water-resistance is a primary design concern. Any cracks that pass through the full thickness of a section are likely to allow some seepage initially, resulting in surface staining and damp patches. Satisfactory performance depends upon autogenous healing of such cracks taking place within a few weeks of first filling in the case of a containment vessel. A crack width limit of 0.2 mm normally applies to all cracks, irrespective of whether or not they pass completely through the section. Where the appearance of a structure is considered to be aesthetically critical, a limit of 0.1 mm is recommended.

2.5 Reinforced Concrete Loadings

The loads (actions) acting on a structure generally consist of a combination of dead (permanent) and live (variable) loads. In limit-state design, a design load (action) is calculated by multiplying

the characteristic (or representative) value by an appropriate partial factor of safety. The characteristic value is generally a value specified in a relevant standard or code. In particular circumstances, it may be a value given by a client or determined by a designer in consultation with the client. In BS 8110 characteristic dead, imposed and wind loads are taken as those defined in and calculated in accordance with BS 6399: Parts 1, 2 and 3. In BS 5400 characteristic dead and live loads are given in Part 2, but these have been superseded in practice by the loads in the appropriate Highways Agency standards.
When EC 2: Part 1.1 was first introduced as an ENV document, characteristic loads were taken as the values given in BS 6399 but with the specified wind load reduced by 10%. This was intended to compensate for the partial safety factor applied to wind at the ULS being higher in the Eurocodes than in BS 8110. Representative values were then obtained by multiplying the characteristic values by factors given in the NAD. In the EN documents, the characteristic values of all actions are given in EC 1, and the factors to be used to determine representative values are given in EC 0. Loadings in reinforced concrete design are classified into the following:

2.5.1 Dead Loads (Permanent Action)

Dead loads include the weights of the structure itself and all permanent fixtures, finishes, surfacing and so on. When permanent partitions are indicated, they should be included as dead loads acting at the appropriate locations. Where any doubt exists as to the permanency of the loads, they should be treated as imposed loads. Dead loads can be calculated from the unit weights given in EC 1: Part 1.1, or from actual known weights of the materials used.

2.5.2 Live Loads (Variable Action)

Live loads comprise any transient external loads imposed on the structure in normal use due to gravitational, dynamic and environmental effects. They include loads due to occupancy (people, furniture, moveable equipment), traffic (road, rail, pedestrian), retained material (earth, liquids, granular), snow, wind, temperature, ground and water movement, wave action and so on. Careful assessment of actual and probable loads is a very important factor in producing economical and efficient structures. Some imposed loads, like those due to contained liquids, can be determined precisely. Other loads, such as those on floors and bridges are very variable. Snow and wind loads are highly dependent on location.

2.6 Review of Reinforced Concrete Design Methods

2.6.1 Elastic Methods

The so-called exact theory of the elastic bending of plates spanning in two directions derives from work by Lagrange, who produced the governing differential equation for plate bending in 1811, and Navier, who in 1820 described the use of a double trigonometric series to analyze freely supported rectangular plates. Pigeaud and others later developed the analysis of panels freely supported along all four edges. Many standard elastic solutions have been produced but almost all of these are restricted to square, rectangular and circular slabs (see, for example, refs. 19, 20 and 21). Exact analysis of a slab having an arbitrary shape and support conditions with a general arrangement of loading would be extremely complex. To deal with such problems, numerical techniques such as finite differences and finite elements have been devised.

2.6.2 Collapse Methods

Unlike in frame design, where the converse is generally true, it is normally easier to analyze slabs by collapse methods than by elastic methods. The most-widely known methods of plastic analysis of slabs are the yield-line method developed by K W Johansen, and the so-called strip method devised by Arne Hillerborg. It is generally impossible to calculate the precise ultimate resistance of a slab by collapse theory, since such elements are highly indeterminate. Instead, two separate solutions can be found – one being upper bound and the other lower bound. With solutions of the first type, a collapse mechanism is first postulated. Then, if the slab is deformed, the energy absorbed in inducing ultimate moments along the yield lines is equal to the work done on the slab by the applied load in producing this deformation. Thus, the load determined is the maximum that the slab will support before failure occurs. However, since such methods do not investigate conditions between the postulated yield lines to ensure that the moments in these areas do not exceed the ultimate resistance of the slab, there is no guarantee that the minimum possible collapse load has been found. This is an inevitable shortcoming of upper-bound solutions such as those given by Johansen's theory.

Conversely, lower-bound solutions will generally result in the determination of collapse loads that are less than the maximum that the slab can actually carry. The procedure here is to choose a distribution of ultimate moments that ensures that equilibrium is satisfied throughout, and that nowhere is the resistance of the slab exceeded. Most of the literature dealing with the methods of

Johansen and Hillerborg assumes that any continuous supports at the slab edges are rigid and unyielding.

2.6.3 Yield Line Analysis Method

Johansen's method requires the designer to first postulate an appropriate collapse mechanism for the slab being considered according to the rules given in section 13.4.2 in Reinforced Concrete Design Manual by (Reynolds and Steedman, 2008). Variable dimensions may then be adjusted to obtain the maximum ultimate resistance for a given load (i.e. the maximum ratio of M/F). This maximum value can be found in various ways, using actual numerical values and employing a trial-and-adjustment process. The work equation may be expressed algebraically and, by substituting various values for the maximum ratio of M/F may be read from a graph relating ó to M/F. Another method is to use calculus to differentiate the equation and then, by setting this equal to zero, determine the critical value of ó.

Yield-line theory is too complex to deal with adequately (Reynolds and Steadman, 2008). Indeed, several textbooks are completely or almost completely devoted to the subject In section 13.4 and Tables 2.49 and 2.50, of Reinforced Concrete Design Handbook by (Reynolds and Steadman, 2008), notes and examples are given on the rules for choosing yield-line patterns for analysis, on theoretical and empirical methods of analysis, on simplifications that can be made by using so-called affinity theorems, and on the effects of corner levers.

2.6.4 Strip Method

Hillerborg devised his strip method in order to obtain a lower-bound solution for the collapse load, while achieving a good economical arrangement of reinforcement. As long as the reinforcement provided is sufficient to cater for the calculated moments, the strip method enables such a lower-bound solution to be obtained. (Hillerborg and others sometimes refer to the strip method as the equilibrium theory; this should not, however, be confused with the equilibrium method of yield-line analysis.) In Hillerborg's original theory (now known as the simple strip method), it is assumed that, at failure, no load is resisted by torsion and thus, all load is carried by flexure in either of two principal directions. The theory results in simple solutions giving full information regarding the moments over the whole slab to resist a unique collapse load, the reinforcement being placed economically in bands. Brief notes on the use of simple strip theory to design rectangular slabs supporting uniform loads are given in section 13.5 and Table 2.51 in Reinforced Concrete Design Handbook by (Reynolds and Steadman, 2008).

However, the simple strip theory is unable to deal with concentrated loads and/or supports and leads to difficulties with free edges. To overcome such problems, Hillerborg later developed his advanced strip method, which involves the use of complex moment fields. Although this development extends the scope of the simple strip method, it somewhat spoils the simplicity and directness of the original concept. A further disadvantage of both Hillerborg's and Johansen's methods is that, being based on conditions at failure only, they permit unwary designers to adopt load distributions that may differ widely from those that would occur under service loads, with the risk of unforeseen cracking. A development that eliminates this problem, as well as overcoming the limitations arising from simple strip theory, is the so-called strip-deflection method due to Fernando and Kemp (ref. 30). With this method the distribution of load in either principal direction is not selected arbitrarily by the designer (as in the Hillerborg method or, by choosing the ratio of reinforcement provided in each direction, as in the yield-line method) but is calculated so as to ensure compatibility of deflection in mutually orthogonal strips. The method results in sets of simultaneous equations (usually eight), the solution of which requires computer assistance (Reynolds and Steedman, 2008).

2.6.5 Limit State Method

In limit state method of reinforced concrete design, the working loads are multiplied by partial factor of safety and the ultimate material strength are divided by further partial factor of safety (Oyenuga, 2011). The limit state design philosophy, which was formulated for reinforced concrete design in Russia during the 1930s, achieves the objectives set out in Section 2.3 51 in Reinforced Concrete Design Handbook by (Reynolds and Steadman, 2008) by considering two types of limit state under which a structure may become unfit for its intended purpose. They include:

2.6.5.1 Serviceability Limit State: This state ensures satisfactory behaviour under service loads. The principal criteria relating to serviceability are the prevention of deflection, vibration or cracking. Under consideration of strength, the bridge members need to be strong enough to withstand the live and dead loads identified above with an adequate margin of safety to allow for uncertainties in loading, material properties and quality of construction and maintenance. Deflection requirement suggest that the reinforced concrete slab should not deflect to an extent that might cause concern or discomfort to users or cause fixed members to become out of plane. Maximum limits for reinforced concrete slab range from span/180 (5.5mm per m of span) to span/360 (2.75mm per of span). A middle value of span/250 (4mm per m of span) is used in Reinforced Concrete Design Handbook by (Reynolds and Steadman, 2008).

2.6.5.2 Ultimate Limit State: This design state ensures that the probability of failure is acceptably low and the structure, or some part of it, is safe for its intended purpose (Oyenuga, 2011).

2.7 Reinforced Concrete Slab

Reinforced concrete slab are plate elements which forms the floor of a structure and are more often subjected to bending (tensile or compressive) but in rare cases (such as bridge deck) subjected to shear (Oyenuga, 2011). Slabs are horizontal members but they can serve as vertical members such walls to infill panels, side walls to drain and sewer appurtenances (Oyenuga, 2011). Slabs can be solid or ribbed, and can span between beams, in either one or two directions,
or be supported directly by columns as a flat slab (Reynord and Steedman, 2008). Slab elements occur also as decking in bridges and other forms of platform structures, and as walling in rectangular tanks, silos and other forms of retaining structures (Reynord and Steedman, 2008).According to (Oyenuga, 2011), there are various types of slab and slab types are influenced by span, use of space which may determine the span, load to be borne by the slab and architectural aesthetics. (Oyenuga, 2011) classified slabs types as solid slabs (cantilever, simply supported and two ways), ribbed floor slab, flat slab and waffle slabs. According to (Sabah, 2017), slabs are divided into suspended slabs, suspended slabs may be divided into two groups: slabs supported on edges of beams and walls and also slabs supported directly on columns without beams and known as flat slabs. Supported slabs may be one-way slabs (slabs supported on two sides and with main reinforcement in one direction only) and two-way slabs (slabs supported on four sides and reinforced in two directions). Where beams are provided in one direction only, the slab is a one-way slab. Where beams are provided in two orthogonal directions, the slab is a two-way slab. However, if the longer side of a slab panel exceeds twice the shorter side, the slab is generally designed as a one-way slab. Flat slab is designed as a one-way slab in each direction. Bending moments and shearing forces are usually determined on strips of unit width for solid slabs, and strips of width equal to the spacing of the ribs for ribbed slabs.

Solid slabs are generally designed as rectangular strips of unit width, and singly reinforced sections are normally sufficient. Ribbed slabs are designed as flanged sections, of width equal to the rib spacing, for sagging moments. Continuous ribbed slabs are often made solid in support regions, so as to develop sufficient resistance to hogging moments and shear forces. Alternatively, in BS 8110, ribbed slabs may be designed as a series of simply supported spans, with a minimum amount of reinforcement provided in the hogging regions to control the cracking. The amount of reinforcement recommended is 25% of that in the middle of the adjoining spans extending into the spans for at least 15% of the span length. The thickness of slabs is normally determined by deflection considerations, which sometimes result in the use of reduced reinforcement stresses to meet code requirements.

2.7.1 Review of Reinforced Concrete Slab Types

2.7.1.1 Flat Slabs

Flat slabs are beamless reinforced concrete slabs supported directly by columns (Oyenuga, 2011).They may be of uniform thickness throughout or have deeper thickness (known as drops) around the vicinity of the carrying columns (Oyenuga, 2011). Flat slabs are used as floor to buildings and as flat roofs to building and reservoir. The stability of flat slabs depends on the monolithic interaction between the columns and the slabs (Oyenuga, 2011). When loaded, a flat slabs deflect in all direction away from the head of the column and the maximum deflection occurs at the middle of the saucer-like depression formed within the panel by the four columns (*Oyenuga, 2011). The columns can be enlarged at the junction of the slabs to form what is referred to as column heads. Column heads may be rectangular or conical in shape. The drops are effective in reducing shearing stress especially where large live loads are involved and because of its enhanced thickness, it provides higher moment of resistance for the negative moment occurring at the column areas. Since flat slab are beamless, they allow both light and circulation within the premises and offer reduced storey height (Oyenuga, 2011). Their formwork are also easy to construct.

2.7.1.2 Ribbed Floor Slab

Ribbed floor slab are also like flat slab used in offices and buildings where large spans are expected. They can be whole concrete ribbed floor or ribbed floor with hollow pots in-fill. A ribbed floor slab is a type of reinforced concrete slab in which some of the volume of concrete in the tension zone is removed and replaced with hollow blocks or left as voids. This reduction in the volume of concrete in the tension zone (below the neutral axis) is based on the assumption that the tensile strength of concrete is zero; hence all the tensile stress is borne by the reinforcement in the tension zone. According to (Oyenuga, 2011), concrete below the neutral axis is theoretically ineffective although, it serve the purpose of resisting the shearing force surrounding the reinforcement bars in tension and connecting the bars to the compression zone. It is therefore advantageous from the point of view of reducing the amount of concrete and thereby saving weight to omit the ineffective concrete while retaining enough to serve the purpose earlier described. The ribbed floor slab may be formed by use of temporary formwork which is removed after construction.

2.7.1.3 Waffle Slab

Waffle slab is an extension of ribbed floor slab in which the slab is ribbed in two directions (Oyenuga, 2011). They are reinforced concrete slab employed for large span construction. Waffle slab are described as the equivalent of two way spanning solid slabs but with ribbed slab system spanning in both direction. Hence, an inverted pot like hollow is formed which serve as the ceiling for the floor below. The design concept of waffle slab is similar to that of ribbed slabs with the difference being that waffle slabs have ribs spanning in both directions and the coefficients used for analyzing the slabs are similar to those used for analyzing two way restrained solid slabs. Waffle slabs are supported on beam or columns, where support zones are made to be uniformly thick.

2.7.1.4 Solid Slabs

Solid slabs are most common in residential areas and offices and are generally employed when the span does not exceed 6m (Oyenuga, 2011). Experience have shown that hen the span exceed 6m, deflection may occur or unnecessarily heavy slab thickness results. Depending on the structural configuration, solid slabs may present itself as cantilever, simply supported slab, continuous slab and two way spanning slabs. One-way slabs generally consist of a series of shallow beams of unit width and depth equal to the slab thickness, placed side by side. Such simple slabs can be supported on brick walls and can be supported on reinforced concrete beams in which case laced bars are used to connect slabs to beams (Sabah, 2017). Two- way spanning slabs on the other hand is a slab with main reinforcement in both directions (Oyenuga, 2011). The reinforcement parallel to the shorter dimension are designed first and act as the main reinforcement while the reinforcement parallel to the longer side are designed later and distributed onto the shorter side reinforcements. It is however recommended that none of the reinforcement sizes should be less than 12mm in diameter except distribution bars which may be 10mm in diameter.

2.8 Serviceability Checks for Reinforced Concrete Slab

2.8.1 Cracking

Cracks in members under service loading should not impair the appearance, durability or water-tightness of a structure. In BS 8110, for buildings, the design crack width is generally limited to 0.3 mm. In BS 5400, for bridges, the limit varies between 0.25 mm and 0.10 mm depending on the exposure conditions. In BS 8007, for structures to retain liquids, a limit of 0.2 mm usually applies. Under liquid pressure, continuous cracks that extend through the full thickness of a slab or wall are likely to result in some initial seepage, but such cracks are expected to self-heal within a few weeks. If the appearance of a liquid-retaining structure is considered aesthetically critical, a crack width limit of 0.1 mm applies.

In EC 2, for most buildings, the design crack width is generally limited to 0.3 mm, but for internal dry surfaces, a limit of 0.4 mm is considered sufficient. For liquid-retaining structures, a classification system according to the degree of protection required against leakage is introduced. Where a small amount of leakage is acceptable, for cracks that pass through the full thickness of the section, the crack width limit varies according to the hydraulic gradient (i.e. head of liquid divided by thickness of section). The limits are 0.2 mm for hydraulic gradients ≥ 5, reducing uniformly to 0.05 mm for hydraulic gradients 35. In order to control cracking in the regions where tension is expected, it is necessary to ensure that the tensile capacity of the reinforcement at yielding is not less than the tensile force in the concrete just before cracking. Thus a minimum amount of reinforcement is required, according to the strength of the reinforcing steel and the tensile strength of the concrete at the time when cracks may first be expected to occur. Cracks due to restrained early thermal effects in continuous walls and some slabs may occur within a few days of the concrete being placed. In other members, it may be several weeks before the applied load reaches a level at which cracking occurs.

Crack widths are influenced by several factors including the cover, bar size, bar spacing and stress in the reinforcement. The stress may need to be reduced in order to meet the crack width limit. Design formulae are given in Codes of Practice in which strain, calculated on the basis of no tension in the concrete, is reduced by a value that decreases with increasing amounts of tension reinforcement. For cracks that are caused by applied loading, the same formulae are used in BS 8110, BS 5400 and BS 8007. For cracks that are caused by restraint to temperature effects and shrinkage, fundamentally different formulae are included in BS 8007. Here, it is assumed that bond slip occurs at each crack, and the crack width increases in direct proportion to the contraction of the concrete.

2.8.2 Deflection

The deflections of members under service loading should not impair the appearance or function of a structure. An accurate prediction of deflections at different stages of construction may also be necessary in bridges, for example. For buildings, the final deflection of members below the support level, after allowance for any pre-camber, is limited to span/250. In order to minimize any damage to non-structural elements such as finishes, cladding or partitions that part of the deflection that occurs after the construction stage is also limited to span/500. In BS 8110, this limit is taken as 20 mm for spans ≥10 m.

The behaviour of a reinforced concrete beam under service loading can be divided into two basic phases: before and after cracking. During the un-cracked phase, the member behaves elastically as a homogeneous material. This phase is ended by the load at which the first flexural crack forms. The cracks result in a gradual reduction in stiffness with increasing load during the cracked phase. The concrete between the cracks continues to provide some tensile resistance though less, on average, than the tensile strength of the concrete. Thus, the member is stiffer than the value calculated on the assumption that the concrete carries no tension. This additional stiffness, known as ‘tension stiffening’, is highly significant in lightly reinforced members such as slabs, but has only a relatively minor effect on the deflection of heavily reinforced members.

2.8.3 Shear

Suspended slabs and foundations are often subjected to large loads or reactions acting on small areas. Shear in solid slabs under concentrated loads can result in punching failures on the inclined faces of truncated cones or pyramids. For design purposes, shear stresses are checked on given perimeters at specified distances from the edges of the loaded area. Where a load or reaction is eccentric with regard to a shear perimeter (e.g. at the edges of a slab, and in cases of moment transfer between a slab and a column), an allowance is made for the effect of the eccentricity. In cases where v exceeds vc, links, bent-up bars or other proprietary products may be provided in slabs not less than 200 mm deep.

In BS 8110, shear reinforcement is required to cater for the difference between the shear force and the shear resistance of the section without shear reinforcement. Equations are given for upright links based on concrete struts inclined at about 45^o, and for bent-up bars where the inclination of the concrete struts may be varied between specified limits. In BS 5400, a specified minimum amount of link reinforcement is required in addition to that needed to cater for the difference between the shear force and the shear resistance of the section without shear reinforcement. The forces in the inclined concrete struts are restricted indirectly by limiting the maximum value of the nominal shear stress to specified values.

In EC 2, shear reinforcement is required to cater for the entire shear force and the strength of the inclined concrete struts is checked explicitly. The inclination of the struts may be varied between specified limits for links as well as bent-up bars. In cases where upright links are combined with bent-up bars, the strut inclination needs to be the same for both.

2.8.4 Torsion

In normal beam-and-slab or framed construction, calculations for torsion are not usually necessary, adequate control of any torsional cracking in beams being provided by the required minimum shear reinforcement. When it is judged necessary to include torsional stiffness in the analysis of a structure, or torsional resistance is vital for static equilibrium, members should be designed for the resulting torsional moment. The torsional resistance of a section may be calculated on the basis of a thin-walled closed section, in which equilibrium is satisfied by a closed plastic shear flow. Solid sections may be modeled as equivalent thin-walled sections. Complex shapes may be divided into a series of sub-sections, each of which is modeled as an equivalent thin-walled section, and the total torsional resistance taken as the sum of the resistances of the individual elements. When torsion reinforcement is required, this should consist of rectangular closed links together with longitudinal reinforcement. Such reinforcement is additional to any requirements for shear and bending.

2.9 Reinforced Concrete Slabs Design Tables and Charts

Design charts and tables are very useful tools for fast determining the percentage of reinforcement for reinforced concrete slabs having known cross-sectional dimensions, characteristic strengths of the concrete and steel, and the ultimate design moment (Bayagoob, Yardim and Ramoda, 2013). They are also useful in the satisfaction of serviceability requirement (deflection and shear) for any structural members with varying cross section. This study will however focus on the development of design tables and charts relevant to design of reinforced concrete reinforced concrete slabs.

2.10 Review of Past Works on Formulation of Design Charts and Tables

(Bayagoob, Yardim and Ramoda, 2013) conducted a study on development of design charts for rectangular section. Simplified design chart has been developed based on BS: 8110 – 97 design rules and several design examples were solved using the developed charts and tables.

Poulos and Small, (2017) carried out a study on development of design charts for concrete industrial slabs. The charts are based on finite element analyses in which the slab is modeled as an elastic plate and the soil is idealized as an equivalent homogeneous isotropic elastic layer of limited depth. The primary design objective is to limit the flexural tensile stresses in the slab to specified allowable values. In developing the design chars, a standard set of parameters has been selected for the main analyses, and then correction factors have been derived to account for the effects of variations from the standard values of soil Young's modulus, soil layer depth, and the loading details. The charts then relate the slab thickness to the maximum flexural tensile stress in the slab.

(Nakov, 2012) developed the design charts for steel fiber reinforced concrete elevated slabs. Determination of the bearing capacity of the slabs was made by using the Yield line theory. Different materials as well as different geometry were considered, making in total four hundred and five (405) combinations. All these combinations were a subject of design by the Yield line theory at the ultimate limit state and design charts with respect of some of the variables was developed.

Guptha and Shanmukesh, (2020) developed the design charts of a parallelogram shaped slab by using the yield line method. Load-deflection behavior of simply supported reinforced concrete parallelogram shaped slabs was analyzed using experimental and analytical methods. Finite element modeling of eighteen numbers of simply supported reinforced concrete parallelogram shaped slabs were made and non-linear analysis carried out on all parallelogram shaped slabs to obtain the load-deflection relationship under the action of uniformly distributed load. To carry out this analysis, a free MS Office (MS Excel sheet) was used. Also, this can be compared with an experimental load deflection curves and using theoretical method, with this results.

The current study will however focus on development of simplified design tables and charts for the analysis and design of reinforced concrete slabs with varying cross sections.

CHAPTER THREE

3.0 MATERIALS AND METHODS

This chapter will present the slab section and detailed procedure employed for the formulation of the design tables and charts for the reinforced concrete slab. The design code of practice used for the formulation of the design tables and charts and also how the design charts can be interpreted during design. Below is a presentation of the aforementioned information.

3.1 Slab Design Details

Two slabs will be considered for design using the developed design table and charts to be presented in chapter four. The respective slabs are to be designed to BS8110: (1997). Internal stresses (maximum bending moment and shear force) have been determined earlier using BS8110: (1997). The slabs will be designed using the design tables and charts developed for reinforced concrete slabs with varying cross section. The design information is presented below:

Table 3.1 Design Details for the Reinforced Concrete Slabs

Design Code	**BS 8110: 1997 (design of concrete structures)**
General Loading Condition	**Specific Density of Concrete = 24KN/m^3** **Characteristic Strength of Concrete = 30N/mm^2** **Characteristic Strength of Steel = 410N/mm^2** **Characteristic Strength of Concrete = 20N/mm^2** **Cover to reinforcement = 25mm**
Factor of Safety	**Dead Load (G_k) = 1.4** **Imposed Load (Q_k) = 1.6** **Ultimate design load = 1.4 G_k + 1.6Q_k**
Design Data	**Fixed End Moment = $\frac{wl2}{12}$** **Free End Moment = $\frac{wl2}{8}$**

	Minimum area of steel = 0.4%bh **For high yield steel =** $\frac{\mathbf{0.15bh}}{\mathbf{100}}$
Slab Details	**Moment to be resisted by concrete is 15.4 and 18.8kNm respectively.** **Width of Slab is 1000mm** **Overall depths of slabs are 150 and 175mm respectively.** **Effective depths of the slabs are 119 and 144mm respectively.**

3.2 Methods for Design of Reinforced Concrete Slabs

The procedure for design of reinforced concrete slabs according to BS 8110: 1997 (design of concrete structures) is presented below:

3.2.1 Ultimate Limit State Design

1 Determine the static system (solid slab, flat slab, cantilever slabs).
2 Choose the cross section (depth and span) for the slabs.
3 Estimate the dead loads (self weight of slabs and finishes, wall loads).
4 Estimate the live loads (load from furniture, equipment and human weight).
5 Analyze the structure and determine the internal stresses (maximum bending moment and shear force).
6 Determine the K-value and if the k-value is less than or equals 0.1569, design the slab for tension reinforcement only but if the k-value exceeds 0.1569 design the slab for tension and compression reinforcements but however compression reinforcement are seldom in slab rather increase the slab cross section.
7 Design the slabs and determine the area of reinforcements.
8 Specify the spacing and diameter of the reinforcing bars.

3.2.2 Serviceability Limit State Checks

1 Check for deflection and where found inadequate, increase either the slab cross section (overall depth) or the area of reinforcing bars.
2 Check for shear stress and design for shear reinforcements but however, shear reinforcement is seldom in slabs except in slab sections with considerable thickness.

3.3 Methods for Formulation of Design Tables and Charts for Reinforced Concrete Slabs

Design tables and charts are valuable for the determination of the area of reinforcement at ultimate limit state design. The design tables and charts will be developed in accordance to BS 8110: 1997 (Design of reinforced concrete structures). The design charts and table to be formulated based on relationship between K represented mathematically as $M/\boldsymbol{F_{cu}}bd^2$ and mμ represented as$100As F_y/bd\boldsymbol{F_{cu}}$. Mathematical model will be derived for obtaining their relationship which will be used as basis for developing the reinforced concrete design charts and tables for the reinforced concrete slabs. Below are procedures for obtaining the mathematical model.

Consider a rectangular section shown in Fig 3.0. In theory of reinforced concrete bending according to Bs 8110: (1997), the concrete is assumed to develop cracks in the region of tensile strain and after cracking, the tension is assumed to be carried by tensile reinforcement. The implication is that the plane section normal to the axis of the member remains plain after bending and the maximum strain in the concrete at the outermost fiber is 0.0035.

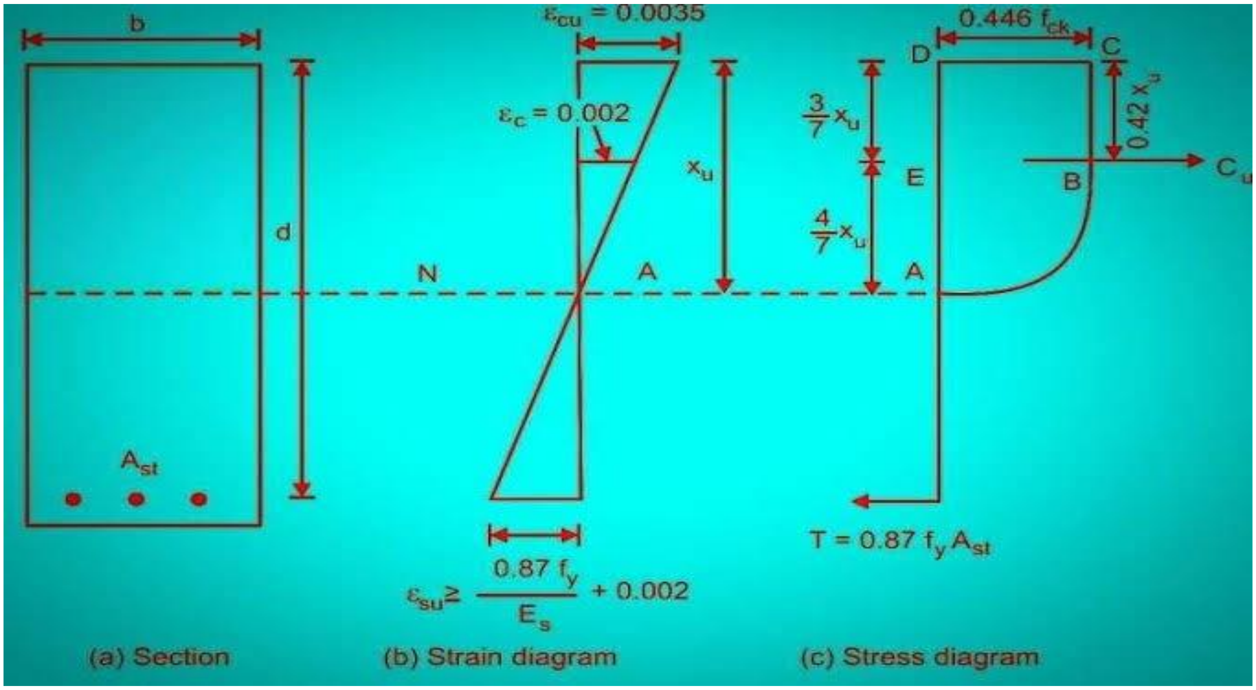

Fig 3.0: Stress to Strain Block Diagram for Singly Reinforced Rectangular Section.

For a singly reinforced section, bending will give rise to resultant compressive strength (force), the equilibrium of the compressive and tensile equation can be written as:

$$0.95F_y\text{As} = 0.45\boldsymbol{F_{cu}}\text{bx}0.9\text{x} \qquad (3.0)$$

Thus the depth of the uniform stress block of the compression concrete (x) is obtained from the above equation

$$\text{x} = 2.346\text{As}F_y/\boldsymbol{F_{cu}}\text{b} \qquad (3.1)$$

The lever arm of the resultant concrete force about the tension steel can be expressed as:

$$\text{z} = \text{d} - 0.45\text{x} \qquad (3.2)$$

Substitute equation (3.1) into equation (3.2) we obtain the relation:

$$\text{z} = \text{d}\,[1 - 1.056\text{As}F_y/\boldsymbol{F_{cu}}\ \text{bd}] \qquad (3.3)$$

Taking the moment about the resultant concrete compression force, the ultimate moment of resistance of the section can be expressed as:

$$\text{Mu} = 0.95F_y\text{AsZ} \quad (3.4)$$

Substitute equation (3.3) into equation (3.4) we obtain the relation:

$$\text{Mu} = 0.95F_y\text{Asd}\,[1 - 1.056\text{As}F_y/\boldsymbol{F_{cu}}\ \text{bd}] \quad (3.5)$$

Dividing both terms of equation (3.5) by Fcubd^2 and using the notations:

$\text{K} = \text{Mu}/\boldsymbol{F_{cu}}\text{bd2}$ $\mu = 100\text{As/bd}$ and $\text{m} = F_y/\boldsymbol{F_{cu}}$ We obtain the following mathematical expression:

$$\text{K} = 0.0095\text{m}\mu - 0.0001(\text{m}\mu)2 \quad (3.6)$$

BS 8110: (1997) stipulates in clause 3.4.4.4 that the lever arm for singly reinforced concrete sections must not exceed 0.95d in order to offer a reasonable concrete area in compression.

Relationship between K and Z is represented by the relation:

$$\text{z/d} - (\text{z/d})2 = \text{k}/0.9 \quad (3.7)$$

At limit of Z=0.95d, therefore the value of k is less than or equal to 0.0428, the steel area should be calculated using Z = 0.95d. Substituting Z = 0.95d in equation (3.4) and using the earlier stated notations (m, μ) obtain the following is obtained:

$$\text{K} = 0.009\text{m}\mu \text{ for K less than or equal to } 0.00428 \quad (3.8)$$

$$\text{K} = 0.0095\ \text{m}\mu - 0.0001(\text{m}\mu)2 \quad (3.9)$$

To ensure uniformity in parameters used in the formulation of the design tables and charts, equation (3.8) can be redefined as:

$$\text{m}\mu = \mathbf{100As}\boldsymbol{F_y}\mathbf{/bd}\boldsymbol{F_{cu}} \quad (3.10)$$

Substituting equation (3.10) into equation (3.8) we have:

$$K = 0.009\left(\frac{100As\boldsymbol{F_y}}{bd\boldsymbol{F_{cu}}}\right) \quad (3.11)$$

Equation (3.11) is a mathematical model used for formulating the design charts and tables for singly reinforced concrete beams, if the value of K is known, the value of ($\mathbf{100As}\boldsymbol{F_y}/\mathbf{bd}\boldsymbol{F_{cu}}$) can be read from the chart and the area of steel required can be computed and specified.

3.3.1 Description and Application of Design Tables and Charts

The design tables comprises of eight columns and a series of rows for values of K, mµ and area of reinforcement required at varying values of effective depth while the design charts is a graphical representation of K and mµ, K value is plotted at the ordinate (y-axis) while the mµ is plotted at the abscissa (x-axis). K value is represented mathematically as $\mathrm{M}/\boldsymbol{F_{cu}}\mathrm{b}d^2$ while the mµ is represented mathematically as $\frac{100\mathrm{As}\boldsymbol{F_y}}{\mathrm{bd}\boldsymbol{F_{cu}}}$, area of steel reinforcement (As) is the output of the design tables and charts formulation process. If the value of the moment, slab width, slabs effective depth, characteristic strength of steel, characteristic strength of concrete is known, then the value of K can be computed and mµ can be read from the graph and thereafter the area of steel reinforcement can be provided.

CHAPTER FOUR

RESULTS AND DISCUSSION

This section presents key findings obtained from the analytical study. Findings includes: developed design charts and tables for design of singly reinforced concrete slab with varying values of $\frac{d}{h}$ ratio at ultimate limit state, method of application of the developed design charts and tables and comparison of the developed design charts and tables and BS 8110 code design equation in order to check for accuracy. Below is a description of the above stated information.

4.1 Presentation of Research Findings

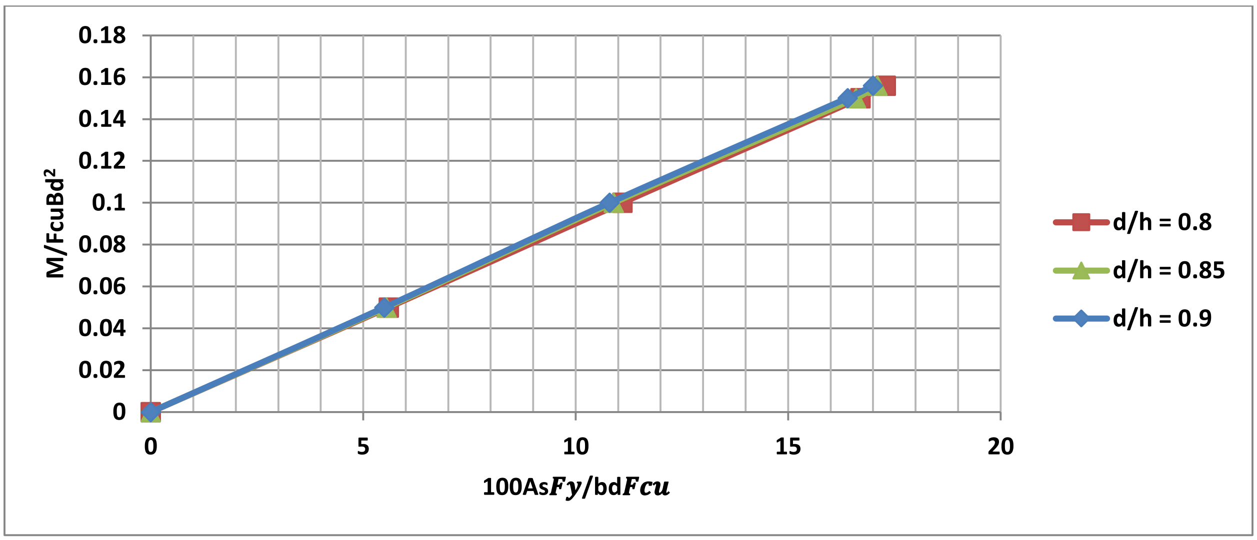

Figure 1Figure 4.1: Developed Slab Design Charts for Singly Reinforced Concrete Slab Design at d/h ratio of 0.8 (Derived from Equation 3.11, Chapter 3).

Table 4.1: Developed Design Table Showing Values for Area of Steel Required (mm^2) at Steel grade (F_y < 410N/mm^2) and Concrete grade (F_{cu} = 20N/mm^2) (Derived from Equation 3.11, Chapter 3).

M/F_{cu}bd^2	100AsF_y/bdF_{cu}	Effective Depth (mm)					
		125	150	175	200	225	250
0.05	5.6	342	410	478	546	615	683
0.06	6.6	404	484	564	644	726	806
0.07	7.8	477	571	666	760	857	951
0.08	8.9	563	674	786	897	1011	1122
0.09	9.9	664	795	928	1058	1193	1479
0.1	11.1	677	812	948	1083	1218	1354
0.11	12.2	799	958	1119	1277	1437	1598
0.12	13.3	943	1130	1320	1507	1696	1885
0.13	14.4	1112	1333	1558	1778	2001	2224
0.14	15.6	1312	1573	1838	2098	2361	2624
0.15	16.7	1548	1856	2169	2475	2786	3096
0.156	17.3	1827	2190	2559	2921	3287	3653

Table 4.2: Developed Design Table Showing Value for Area of Steel Required (mm^2) at Steel grade (F_y< 410N/mm^2) and Concrete grade (F_{cu}= 25N/mm^2) (Derived from Equation 3.11, Chapter 3).

M/F_{cu}bd^2	100AsF_y/bdF_{cu}	Effective Depth (mm)					
		125	150	175	200	225	250
0.05	5.6	427	512	598	683	768	854
0.06	6.6	504	604	706	806	906	1008
0.07	7.8	595	713	833	951	1069	1189
0.08	8.9	702	841	983	1122	1261	1403
0.09	9.9	828	992	1160	1324	1488	1656
0.1	11.1	846	1015	1184	1354	1523	1692
0.11	12.2	998	1198	1397	1598	1797	1997
0.12	13.3	1178	1414	1648	1886	2120	2356
0.13	14.4	1390	1669	1945	2225	2502	2780
0.14	15.6	1640	1969	2295	2626	2952	3280
0.15	16.7	1935	2323	2708	3098	3890	3870
0.156	17.3	2283	2741	3195	3655	4590	4567

Table 4.3: Developed Design Tables Showing Values for Area of Steel Required (mm²) at Steel grade (F_y = 460N/mm²) and Concrete grade (F_{cu}= 20N/mm²) (Derived from Equation 3.11, Chapter 3).

$M/F_{cu}bd^2$	$100AsF_y/bdF_{cu}$	Effective Depth (mm)					
		125	150	175	200	225	250
0.05	5.6	304	365	426	487	548	609
0.06	6.6	356	431	503	575	647	719
0.07	7.8	420	509	594	679	763	848
0.08	8.9	496	601	701	801	900	1001
0.09	9.9	585	709	827	945	1062	1181
0.1	11.1	603	724	845	965	1086	1207
0.11	12.2	712	854	997	1139	1281	1424
0.12	13.3	840	1008	1176	1344	1512	1680
0.13	14.4	991	1189	1388	1585	1784	1982
0.14	15.6	1169	1403	1637	1870	2105	2339
0.15	16.7	1379	1656	1932	2206	2483	2760
0.156	17.3	1627	1954	2280	2603	2930	3257

Table 4.4: Developed Design Tables Showing Values for Area of Steel Required (mm²) at Steel grade (F_y = 460N/mm²) and Concrete grade (F_{cu}= 25N/mm²) (Derived from Equation 3.11, Chapter 3).

$M/F_{cu}bd^2$	$100AsF_y/bdF_{cu}$	Effective Depth (mm)					
		125	150	175	200	225	250
0.05	5.6	380	457	533	609	685	761
0.06	6.6	448	539	629	719	808	898
0.07	7.8	529	636	742	848	953	1060
0.08	8.9	624	750	876	1001	1125	1251
0.09	9.9	736	885	1034	1181	1328	1476
0.1	11.1	754	905	1056	1207	1357	1508
0.11	12.2	890	1068	1246	1424	1601	1779
0.12	13.3	1050	1260	1470	1680	1889	2099
0.13	14.4	1239	1487	1734	1982	2229	2477
0.14	15.6	1462	1960	2046	2339	2630	2887
0.15	16.7	1725	2313	2414	2760	3103	3406
0.156	17.3	2036	2729	2849	3257	3662	4019

Table 4.5: Developed Design Charts and BS 8110 Solution Comparison.

Design Examples	Chart Solution	BS 8110 Solutions	Error (%)
	As (mm^2)	As (mm^2)	(%)
1	354	353	0.28
2	360	357	0.84

Table 4.6: Developed Design Table and BS 8110 Solution Comparison.

Design Examples	Table Solution	BS 8110 Solutions	Error (%)
	As (mm^2)	As (mm2)	(%)
1	358	353	1.41
2	362	357	1.40

Table 4.7: Deflection Check for Singly Reinforced Concrete Slab at Different Support Condition and Varying Effective Depth

Support Conditions	Basic span/ Effective Depth Ratio	Effective Depth	Length of Shorter Span	Modification Factor	Remark
Cantiliver	7	≤125mm	≤1500mm	≤ 2	Satisfied
Simply Supported	20	≤125mm	≤1500mm	≤ 2	Satisfied
Continuous	26	≤125mm	≤1500mm	≤ 2	Satisfied
Cantiliver	7	≤150mm	≤2000mm	≤ 2	Satisfied
Simply Supported	20	≤150mm	≤2000mm	≤ 2	Satisfied
Continuous	26	≤150mm	≤2000mm	≤ 2	Satisfied
Cantiliver	7	≤150mm	≤5000mm	≤ 2	Not satisfied, increase depth of section

Simply Supported	**20**	**≤150mm**	**≤5000mm**	**≤ 2**	**Satisfied**
Continuous	**26**	**≤150mm**	**≤5000mm**	**≤ 2**	**Satisfied**

The developed design charts and tables were based on the mathematical derivation in Chapter three (equation 3.11). The design charts and tables were developed in accordance with BS 8110: 1997 (Design of Reinforced Concrete Structures). The design charts is a graphical representation of K ($\frac{M}{F_{cu}}\mathrm{b}d^2$) against mμ ($100\mathrm{As}F_y/\mathrm{bd}F_{cu}$) at varying values of $\frac{d}{h}$. K value is plotted at the ordinate (Y-axis) while the mμ value is plotted at the abscissa. The developed design table shows the value of area of steel required for different values of effective depth. Either the developed design charts or tables could be applicable to design of singly reinforced concrete slab at ultimate limit state.

The serviceability limit check for deflection as depicted in Table 4.7 was based on the effective depth, length of the shorter span and modification factor.

4.2 Design of Singly Reinforced Concrete Slab

In order to examine the validity of the developed design charts and tables, design examples for two reinforced concrete slab have been solved using the developed design charts and tables and the design output were compared to that obtained using BS 8110 equations. The design examples are presented below:

Steel characteristic strength = 410N/mm^2

Concrete characteristic strength = 20N/mm^2

Design Moments = 15.4kNm and 18.8kNm

Overall depths of slab are 150mm and 175mm respectively.

Breadth of Slab = 1000mm

Cover to reinforcement = 25mm

4.2.1 Design of Slab Using Developed Charts and Tables

Slab section with shorter span of 2500mm, moment of 15.4kNm and overall depth of 150mm

Effective depth (d) = h – ó/2 – c = 150 – 6- 25 = 119mm

$\frac{d}{h} = \frac{119}{150}$ = 0.79 = 0.8

The developed design charts to be used is that presented in Figure 4.1

K = $\frac{M}{F_{cu}bd^2} = \frac{15.4\text{x}10^6}{20\ x\ 1000x119x119}$ = 0.054

Using the value of K calculated (0.054) at a $\frac{d}{h}$ ratio of 0.8, the value of mµ at the abscissa (x-axis) corresponding to the value of K (0.054) as depicted in Figure 4.1 is 6.1.

Therefore, determining the area of reinforcement required (As).

Using the developed design charts (Figure 4.1) $100AsF_y/bdF_{cu}$ = 6.1.

Therefore, As = $6.1x1000x119x20/100x410$ = 354mm^2

Area of Steel required for slab section having a moment of 15.4kNm is 354mm^2

Area of Steel provided for slab section having a moment of 15.4kNm is 556mm^2 (Y12@ 200mmc/c).

Using the developed design table, value for area of steel required at an effective depth of 119mm and concrete and steel grade of 20 and 410N/mm^2 can be read from Table 3.1.

Using the developed design table (Table 4.1), area of steel required = 358mm^2.

Area of Steel provided for slab section having a moment of 15.4kNm is 556mm^2 (Y12@ 200mmc/c).

Check for Deflection

Basic span/effective depth ratio = $\frac{20+20}{2}$ = 20 (both end simply supported)

$\frac{100As}{bd}$ = $\frac{100x556}{1000x144}$ = 0.39%

Therefore, modification factor = 1.95

With a modification factor of 1.92, basic span/effective depth ratio of 20, length of shorter span of 2500mm, the deflection is said to be satisfied as depicted in Table 4.7.

Slab Section with length of shorter span of 3500mm, moment of 18.8kNm and overall depth of 175mm

Effective depth (d) = h – ó/2 – c = 175 – 6- 25 = 144mm

$\frac{d}{h}$= $\frac{144}{175}$ = 0.85

The developed design charts to be used is that presented in Figure 4.1.

K = $\frac{M}{F_{cu}bd^2}$ = $\frac{18.8\text{x}10^6}{20\ x\ 1000x144x144}$ = 0.045

Using the value of K calculated (0.045) at a $\frac{d}{h}$ ratio of 0.85, the value of mµ at the abscissa (x-axis) corresponding to the value of K (0.045) as depicted in Figure 4.1 is 5.12.

Therefore, determining the area of reinforcement required (As).

Using the developed design charts (Figure 4.1) $100AsF_y/bdF_{cu}$ = 5.12.

Therefore, As = $5.12x1000x144x20/100x410$ = 360mm^2

Area of Steel required for slab section having a moment of 18.8kNm is 360mm^2

Area of Steel provided for slab section having a moment of 15.4kNm is 646mm^2 (Y12@ 175mmc/c).

Using the developed design table, values for area of steel required at an effective depth of 144mm and concrete and steel grade of 20 and 410N/mm^2 can be read from Table 4.1.

Using the developed design table (Table 4.1), area of steel required = 362mm^2.

Area of Steel provided for slab section having a moment of 15.4kNm is 646mm^2 (Y12@ 175mmc/c).

Check for Deflection

Basic span/effective depth ratio = $\frac{20+20}{2}$ = 20 (both end simply supported)

$\frac{100As}{bd} = \frac{100x646}{1000x144}$ = 0.45%

Therefore, modification factor = 2

With a modification factor of 2, basic span/effective depth ratio of 20, length of shorter span of 3500mm, the deflection is said to be satisfied as depicted in Table 4.7.

4.2.2 Design of Slab Using Code of Practice (BS 8110)

Slab section with shorter span of 2500mm, moment of 15.4kNm and overall depth of 150mm

Effective depth (d) = h – ó/2 – c = 150 – 6- 25 = 119mm

K = $\frac{M}{F_{cu}bd^2} = \frac{15.4\text{x}10^6}{20\ x\ 1000x119x119}$ = 0.054

From Table 10.11 (Reinforced Concrete Design by Oyenuga, 2011)

From K value (0.054), lever arm (La) = 0.94.

As= $\frac{15.4x10^6}{0.95\ x\ 410x0.94x119}$ = 353mm^2

Therefore, Area of Steel required is 353mm^2

Area of steel provided = 566mm^2

Check for Deflection

Basic span/effective depth ratio = $\frac{20+20}{2}$ = 20 (both end simply supported)

$\frac{100As}{bd} = \frac{100x556}{1000x144}$ = 0.39%

Therefore, modification factor = 1.95

With a modification factor of 1.92, basic span/effective depth ratio of 20, length of shorter span of 2500mm, the deflection is said to be satisfied as depicted in Table 4.7.

Slab Section with length of shorter span of 3500mm, moment of 18.8kNm and overall depth of 175mm

Effective depth (d) = h – ó/2 – c = 175 – 6- 25 = 144mm

$K = \frac{M}{F_{cu}bd^2} = \frac{18.8x10^6}{20 \text{ x } 1000x144x144}$ = 0.045

From Table 10.11 (Reinforced Concrete Design by Oyenuga, 2011)

From K value (0.045), lever arm (La) = 0.94.

$As = \frac{18.8x10^6}{0.95 \text{ x } 410x0.94x144}$ = 357mm^2

Therefore, Area of Steel required is 357mm^2

Area of Steel provided for slab section having a moment of 15.4kNm is 556mm^2 (Y12@ 200mmc/c).

Check for Deflection

Basic span/effective depth ratio = $\frac{20+20}{2}$ = 20 (both end simply supported)

$\frac{100As}{bd} = \frac{100x566}{1000x144}$ = 0.45%

Therefore, modification factor = 1.92

With a modification factor of 1.92, basic span/effective depth ratio of 20, length of shorter span of 3500mm, the deflection is said to be satisfied as depicted in Table 4.7.

4.3 Check for Accuracy of Developed Design Chart

To check the accuracy of the developed design charts, the slab design examples solved using BS 8110 code design equations will be compared with that solved using the developed design charts. Deviation in values of the design output will be calculated and expressed as error in percentages.

Error is calculated as $\frac{\text{Estimated Value}-\text{Actual Value}}{\text{Actual Value}}$ x 100

Slab section with moment of 15.4kNm and overall depth of 150mm

Estimated value (value obtained using developed design charts) = 354mm^2

Actual value (value obtained using BS 8110 Solution) = 353mm^2

Error (%) is calculated as $\frac{354-353}{353}$ x 100 = 0.28

Slab Section with moment of 18.8kNm and overall depth of 175mm

Estimated value (value obtained using developed design charts) = 360mm^2

Actual value (value obtained using BS 8110 Solution) = 357mm^2

Error (%) is calculated as $\frac{360-357}{357}$ x 100 = 0.84

4.3 Check for Accuracy of Developed Design Tables

Slab section with moment of 15.4kNm and overall depth of 150mm

Estimated value (value obtained using developed design charts) = 358mm^2

Actual value (value obtained using BS 8110 Solution) = 353mm^2

Error (%) is calculated as $\frac{358-353}{353}$ x 100 = 1.41

Slab Section with moment of 18.8kNm and overall depth of 175mm

Estimated value (value obtained using developed design charts) = 360mm^2

Actual value (value obtained using BS 8110 Solution) = 362mm^2

Error (%) is calculated as $\frac{362-357}{357}$ x 100 = 1.40

From outcome obtained from accuracy check for developed design charts and tables as presented in Table 4.5 and 4.6 respectively, it can be deduced that the developed design charts is more reliable in application for structural design of singly reinforced concrete slab than the developed design tables. This is due to the difference in the value of error estimated as error computed for the developed design charts is negligible (less than 1%) compared to that of the developed design tables (error greater than 1%). The discrepancy in value for area of steel calculated using the developed design tables and charts and BS 8110 design equations could be attributable to subjective reading of the developed design charts and tables. Ultimately, a reasonable agreement between the developed design tables and charts solution and BS 8110 design solutions is observed as depicted in Table 4.5 and 4.6 respectively.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

From the findings obtained from formulation of design tables and charts for design of singly reinforced concrete slab with varying cross section, the following deduction can be made:

1 The developed design charts and tables for design of singly reinforced concrete slab were developed for a wide range of geometrical parameters expressed as non dimensional parameter.
2 The design charts and tables was developed using a mathematical derivation.
3 The procedure for application of the developed design charts and tables was simple and straight forward.
4 The use of non dimensional parameter for developing the design charts and tables constrained the developed design charts and tables to singly reinforced concrete slab.
5 The use of $\frac{d}{h}$ ratio in developing the design charts increased the number of alternatives to be considered in the design of singly reinforced concrete slab.
6 The developed design charts was relatively reliable compared to the developed design tables.
7 A reasonable agreement exists between the value for area of reinforcement for the developed design charts and tables and BS 8110 design solution.
8 The developed design charts and tables were adjudged to be effective, efficient and structurally reliable.

5.2 Recommendation

Based on the outcome of the analytical study on formulation of design tables and charts for singly reinforced concrete slab, the following recommendations can be made:

1. The developed design charts and tables are applicable to singly reinforced concrete slab with varying values of effective depth to overall depth ratio.
2. The developed design charts and tables provide a reliable solution for design of singly reinforced concrete slab.
3. The developed design charts and tables is valuable to undergraduate and postgraduate students of tertiary institution, lecturers and practicing engineers at both field and design offices.
4. Painstaking conservative judgment must be applied in the interpretation of the developed design charts and tables in order to minimize or avoid design flaws.

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APPENDICES

APPENDIX A

Sample Design of Reinforced Concrete Slab

The slab to be considered for design is a two way slab with length on the shorter side as 5225mm and length on the longer side as 5725mm and contains additional partition loads. The design of the slab is presented below:

Member Reference	Calculations	Outputs
	Loadings	
	Concrete own load = thickness of slab x specific	
Clause 10.1,	weight of concrete = 0.15 x 24 = 3.60kN/m^2	
Chapter 10,	Finishes = 1.2kN/m^2	
Reinforced	Partition allowance = 1kN/m^2	
Concrete Design by	Total dead load (Gk) = 5.8kN/m^2	
Oyenuga, (2011)	Live load (Qk) = 1.5kN/m^2	
	Hence, ultimate design load = 1.4Gk + 1.6Qk	
	= 1.4x5.8 + 1.6x1.5 = 10.52kN/m^2	10.52kN/m^2
	Effective depth (d) = Overall depth – half area of reinforcement – cover to reinforcement = 150 – 12/2 – 20 = 124mm	
	Design of Slab at Ultimate Limit State	
	Two way slab with length on shorter side (Lx) = 5725mm and length on longer side (Ly) = 5225mm	
	L_y/L_x = 5725/5225 = 1.1<2.0	

	To be designed as two way slab. Using the moment coefficient table for restrained solid slabs.	To be designed as two way slab
Table 10.8, Reinforced Concrete Design by Oyenuga, (2011).	Short span coefficients = -0.056 and 0.042 Long span coefficient = -0.045 and 0.032 Additional Partition loads. = weight of block wall plus finishes x factor of safety height of wall x length and width of the slab added. = 3.47x 1.4x 3 x (5.725+ 5.225) = 159.585kN This can be assumed to be uniformly distributed over the entire area of 5.725x 5.225m Hence, the uniformly distributed load = 159.585KN/(5.725x5.225) kN/m^2 = $5.335kN/m^2$ Total uniformly distributed load = 5.335 + 10.52 = $15.855kN/m^2$	$15.855kN/m^2$
	Short Span Design **Design of Mid-Span** M = ßy x Mx Lx = 0.042 x 15.855x 5.225 = 18.18kNm	
Table 10.11, Reinforced Concrete Design by Oyenuyga, (2011)	K = 18.18 x $\frac{106}{20 \text{ x } 1000\text{x } 124 \text{ x } 124}$ = 0.059< 0.156 For K value of 0.059, lever arm (la) = 0.93 As = M/0.95FyZ where Z = lever arm (la) x effective depth (d)	Design for only tension reinforcements.
	Therefore, As = 18.18 x $\frac{106}{0.95 \text{ x } 410\text{x } 0.93\text{x}124}$ = $405mm^2$ Provide Y12@ 200mmc/c (As = $566mm^2$)	Provide Y12@ 200mmc/c (As = $566mm^2$)
	Design of Continuous Edge (Support) Moment = 0.005x 15.855x 5.225 = 24.35kNm	

Table 10.3, Reinforced Concrete Design by Oyenuga, (2011)	$K = 24.35 \times \frac{106}{20 \times 1000 \times 124 \times 124} = 0.079 < 0.156$ For K value of 0.079, lever arm (la) = 0.93 Therefore, $As = 24.35 \times \frac{106}{0.95 \times 410 \times 0.93 \times 124} =$ $542mm^2$ Provide Y12@ 175mmc/c ($As = 646mm^2$)	Design for only tension reinforcements. Provide Y12@ 175mmc/c ($As = 646mm^2$)
	Long Span **Design of Mid Span** Effective Depth = 124mm – 12mm = 112mm Moment = 0.034x 15.855x 5.225 = 14.717kNm	
Table 10.3, Reinforced Concrete Design by Oyenuga, (2011)	$K = 14.72 \times \frac{106}{20 \times 1000 \times 112 \times 112} = 0.059 < 0.1569$ For K value of 0.059, lever arm (la) = 0.93 Therefore, $As = 14.72 \times \frac{106}{0.95 \times 410 \times 0.93 \times 112} =$ $363mm^2$ Provide Y12@ 200mmc/c ($As = 566mm^2$)	Design for only tension reinforcements. Provide Y12@ 200mmc/c ($As = 566mm^2$)
	Design of Continuous Edge (Support) Moment = 0.045x 15.855x 5.225 = 19.48kNm	
Table 10.3, Reinforced Concrete Design by Oyenuga, (2011)	$K = 19.48 \times \frac{106}{20 \times 1000 \times 112 \times 112} = 0.078 < 0.1569$ For K value of 0.078, lever arm (la) = 0.91 Therefore, $As = 19.48 \times \frac{106}{0.95 \times 410 \times 0.91 \times 112} =$ $491mm^2$ Provide Y12@ 200mmc/c ($As = 566mm^2$)	Design for only tension reinforcements. Provide Y12 @ 200mmc/c ($As = 566mm^2$)
Table 10.9, Reinforced Concrete Design by Oyenuga, (2011)	**Serviceability Limit State Checks** Deflection check is limited to the Shorter Span Basic Span to Effective Depth Ratio = 23+23/2 =23	

	$\frac{100As}{bd}$ = 100x 566/1000x124 = 0.45% With $\frac{100As}{bd}$ of 0.45%, modification factor (M.F) becomes 1.87 Deflection Required = Shorter Span/23xm.f = 5225/23x1.87 = 121.44 < 124mm. Therefore deflection requirement is satisfied. **Shear Checks** Seldom in slab except slab section thicker than 200mm. **Cracking** For slab less than or equal to 150mm in thickness and steel grade greater than or equal to 410N/mm^2 No check is required but the clear distance between bars must not exceed 3d where d is the effective depth of the slab. Clear distance = 3x 150mm = 450mm Spacing of bar = 200mm – 12mm = 188mm Therefore, 188mm<450mm crack requirement is satisfied.	Deflection requirement satisfied. Shear Requirement Satisfied. Crack Requirement satisfied.