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Buckling Analysis of thick Isotropic Rectangular SSSS plates using Ritz energy method in Polynomial displacement functions

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Abstract: This research study focuses on the buckling analysis of elastic thick isotropic rectangular plates with simply-supported (SSSS) edges. The Ritz energy method was employed using polynomial displacement functions to determine the critical buckling load parameters of the plate under uniaxial in-plane compressive load. The direct governing equation for the plate was derived using orthogonal polynomial displacement functions and a polynomial shear deformation function. The equation was further analyzed to obtain three simultaneous governing equations for the determination of displacement coefficients (j_1, j_2, j_3). The results of the study showed that the polynomial displacement function for SSSS rectangular thick plates provides a simple and efficient solution for determining stiffness coefficient values based on SSSS plate boundary conditions. The method was found to be very close compared with other researchers' works, and the results had very simple mathematical applications. This study contributes to the understanding of buckling behaviour in thick isotropic rectangular plates and provides a valuable tool for future research in this area.

***KEYWORDS*:** *Thick plates, displacement functions, in-plane-displacements, out-of-plane displacement, shear rotation, buckling and rectangular, Ritz energy method*

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I INTRODUCTION

Plates consist of two parallel faces, commonly known as structural members, that are initially flat. These faces are separated by the plate's thickness (h), according to Gwarah (2019). Plate theory assumes that the plate's thickness is small compared to its dimensions (length, width, diameter, etc.), and that plate boundaries are either straight or curved lines. An SSSS plate refers to a plate that is simply supported along all four edges.Plate structures find application in numerous Engineering disciplines, including Mechanical Engineering, Automotive Industries, Aeronautical Engineering, Military, and Structural Engineering, according to Onyeka (2022). In the field of Structural Engineering, plates are extensively utilized in various applications, such as roof and floor slabs, bridge decks, foundation footings, water tanks, ship hulls, and spacecraft panels, as reported by Onyeka and Okeke (2021). Toachieve the properties of plates, good knowledge of their structural behaviour and failure conditions for safer and more economical design isessential(Chukwudi, 2022).

Plates are typically exposed to transverse, compressive, and occasionally dynamic loads acting in the middle plane of the plate (Ezeh, Onyechere, Ibearugbulem, Anya, and Anyaogu, 2018). Such loads can lead to plate buckling under certain conditions, which is of great practical importance (Sachin, Rajesh, Atteshamuddin, and Manas, 2015).

Analyzing a plate as a three-dimensional problem is technically correct (Chukwudi and Okafor, 2022). However, the mathematical complexities involved reduce this problem to a two-dimensional or one-dimensional problem through certain simplifying assumptions (Onyechere, 2019). Thick plate buckling belongs to the class of three-dimensional problems. Plate theories are essential in plate analysis because they are used to evaluate deformations and stresses in a plate subjected to loads (Mokhtar, Addelaziz, and Noureddine, 2016). These theories include thin plate theories and thick plate theories.

The classical plate theory (CPT), also known as the thin plate theory, is the most popular theory that neglects the transverse shear deformation effects. The CPT theory provides good results for thin plates (Thai and Kim, 2015). However, it underestimates the deflections and overestimates the natural frequencies as well as buckling loads of moderately thick and thick plates (Omid, Saeid, and Mojtaba, 2017). To mitigate this error in CPT, a number of shear deformation plate theories have been developed to take the effects of shear deformation into the analyses of plates (Reissner, 1945). These include the First Order Shear Deformation Theories (FSDT), also known as the Mindlin plate theory, and High Order Shear Deformation Theories (HSDT) (Reddy, 2004).

In 1951, Mindlin developed a displacement-based first-order shear deformation theory (FSDT) that considers the effect of transverse shear deformation by assuming a linear variation of in-plane displacements across the plate's thickness (Ezeh, Onyechere, and Anya, 2018). The theories do not satisfy the zero traction boundary conditions on the top and bottom surfaces of the plate, thus requiring a shear correction factor to satisfy the constitutive relations for transverse shear stresses and strains (Rajesh and Meera, 2016). The weaknesses observed by the Classical Plate Theory (CPT) and the first-order shear deformation theory (FSDT) gave rise to the development of higher-order shear deformation Theories (HSDT) (Uchechi and Ibearugbulem, 2019).Several plate theories, called higher-order shear Deformation Plate Theories (HSDTs), have been developed to obtain a more realistic variation of the transverse shear strains and stresses across the thickness of a plate. These theories assume a parabolic shear strain distribution across the plate thickness, which can be higher-order parabolic functions. Some examples of these theories include the ones proposed by Reddy (1984), Murthy (1981), Karama, Afaq, and Mistou (2009), Mantari, Oktem, and Guedes (2012), Zenkour (2006), Mechab, Atmane, Tounsi, Belhadj, and Bedia (2010), Touratier (1991), and Badaghi and Saidi (2010), among others.The primary objective of developing these theories was to provide a method that would overcome the limitations of first-order shear deformation theories (FSDT) and accurately account for the effect of shear deformation without using shear correction factors. By incorporating transverse strains and stresses in the formulation, HSDTs can provide more reliable solutions for the analysis of thick plates in bending and buckling.

Several research studies have been conducted to analyze thick plates using HSDTs without using the shear correction factor. For instance, Gunjal, Hajare, Sayyad, and Ghodle (2015) used a refined trigonometric shear deformation function to analyze thick isotropic square and rectangular plate buckling. Matsunaga (1994) treated the out-of-plane buckling problems of plates subjected to in-plane stresses. Ghugal (2012) used exponential shear deformation theory to analyze the buckling of thick isotropic plates subjected to uniaxial and biaxial in-plane forces. Abdollah, Bahram and Javad (2016) worked on the analysis of a 3-D elasticity buckling solution for simply supported thick rectangular plates, while Badaghi and Saidi (2010) used the Levy-type solution for buckling analysis of thick functionally-graded rectangular plates based on the Higher-Order Shear Deformation Plate Theory. However, the mathematical formulations for thick plate analysis using HSDTs are often very complex to handle.

Upon reviewing the existing literature, it is evident that none of the authors have conducted research on the use of polynomial displacement functions in the Ritz energy method for analyzing thick plates. Rather, the studies have primarily relied on techniques such as double Fourier series, exponential shear deformation functions, and generalized differential quadrature. However, the complexity associated with using double Fourier series for thick plate analysis has led engineers to resort to thin plate analysis, despite its challenges. The classical plate theory (CPT) is a popular option due to its simplicity, but it neglects the effects of transverse shear deformation and is only suitable for analyzing thin plates. Consequently, this research aims to tackle these issues by employing the Ritz energy method with polynomial displacement functions to analyze buckling in thick, isotropic SSSS rectangular plates.

II. MATERIALS AND METHODS

A. Formulation of Direct Governing Equation for Thick Plate Analysis

In this study, the researcher is examining a rectangular plate depicted in Fig.2.1. The plate has a length of ‘a’ in the x-direction, ‘b’ in the y-direction, and a thickness of ‘h’ in the z-direction. We assume that the z-direction is positive when facing downward. The plate occupies an area in a Cartesian coordinates system where $0 \leq x \leq a$, $0 \leq y \leq b$, and $-h/2 \leq z \leq h/2$. The aspect ratio is determined by b/a. Non-dimensional coordinates $R = x/a$ in the x-direction and $Q = y/b$ in the y-direction occupy the domain $0 \leq R \leq 1$ and $0 \leq Q \leq 1$. The study of buckling in plates is a crucial aspect of plate analysis, as it helps to identify the critical buckling loads of a plate.

In order to thoroughly understand this analysis, the researchers exploring the mathematical formulation in greater depth. The first step is to utilize the theory of elasticity in plate analysis to identify a variety of plate elements. These elements are then condensed into a Total Potential Energy functional, which ultimately allows the researcher to establish the Direct Governing Equation for the thick plate.

Total Potential Energy

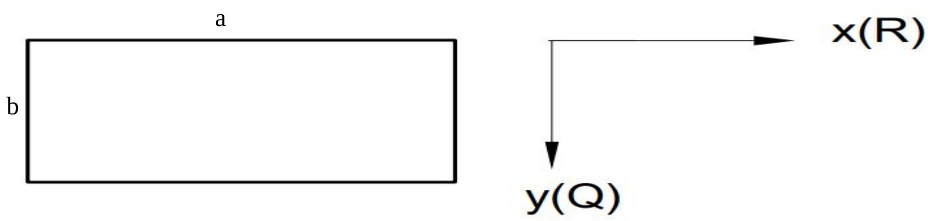

Fig. 2.1: SSSS rectangular plate

The total potential energy, Π, is mathematically determined in Equation (2.1):

$$\Pi = U + \Omega \quad (2.1)$$

$$U = \int_x \int_y \left[\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma \cdot \varepsilon dz \right] dxdy \quad (2.2)$$

$$\Omega = Nx/2 \int_x \int_y (\partial w/\partial x)^2 \partial x \partial y \quad (2.3)$$

Where:

Π = the total potential energy of the system.

U = strain energy of deformation (the potential of internal forces).

Ω = potential energy of external forces (the potential of external forces).

N_x = axial compressive load applied at the mid-plane of the plate.

$$\rho = a/h \quad (2.4)$$

$$\beta = b/a \ and \ b = a\beta \quad (2.5)$$

a is the length of the plate along the x-axis.

b is the breadth of the plate along the y-axis, h the thickness of the plate along the z-axis.

Integrating the total potential energy functional within the intervals of the non-dimensional parameters in the x and y directions, Equation (2.6) is obtained.

$$\Pi = \frac{abD}{2a^4} \int_0^1 \int_0^1 [q_1 j_1^2 - 2q j_1 j_2 + q_3 j_2^2] \left(\frac{d^2H}{dR^2}\right)^2 + \frac{1}{\beta^2} [2q_1 j_1^2 - 2q_2 j_1 j_2 - 2q_2 j_1 j_3] \left(\frac{d^2H}{dRdQ}\right)^2 + \frac{(1+v)}{\beta^2} g_3 c_2 c_3 \left(\frac{d^2H}{dRdQ}\right)^2 + \frac{(1-v)}{2\beta^2} [q_3 j_2^2 + q_3 j_3^2] \left(\frac{d^2H}{dQ^2}\right)^2 + \frac{(1-v)\rho^2}{2} q_4 j_2^2 \left(\frac{dH}{dR}\right)^2 + \frac{(1-v)\rho^2}{2\beta^2} q_4 j_3^2 \left(\frac{dH}{dQ}\right)^2 \Bigg] dRdQ$$

$$-\frac{Nx}{2} \int_0^1 \int_0^1 \left(\frac{\partial w}{\partial x}\right)^2 dRdQ \quad (2.6)$$

Direct Governing Equation

To derive the governing equation for the thick plate, there is need to minimize the potential energy functional by differentiating it with respect to the three coefficients of displacement *J_1, J_2, and J_3*. The three equations obtained simultaneously from this process represent the direct governing equation of the thick plate.

$$\frac{d\Pi}{dj_1} = \frac{d\Pi}{dj_2} = \frac{d\Pi}{dj_3} = 0 \qquad (2.7)$$

Substituting Equation (2.6) into Equation (2.7), Equations (2.8) to (2.10) are obtained.

$$\frac{d\Pi}{dJ_1} = \frac{D}{a^4}\int_0^1\int_0^1 [J_1 + q_2 j_2]\left(\frac{d^2H}{dR^2}\right)^2 + \frac{1}{\beta^2}[2q_1 j_1 + q_2 j_2 + q j_3]\left(\frac{d^2H}{dRdQ}\right)\Bigg] dxdy^2$$

$$-\int_0^1\int_0^1 \frac{dFF}{dc_1} dRdQ = 0 \qquad (2.8)$$

$$\frac{d\Pi}{dj_2} = \frac{D}{a^4}\int_0^1\int_0^1 [q_3 j_2 - q_2 j_1]\left(\frac{d^2H}{dR^2}\right)^2 + \frac{1}{\beta^2}[-q_2 j_1]\left(\frac{d^2H}{dRdQ}\right)^2$$

$$+\frac{(1-v)}{\beta^2}\left[\frac{1}{2}q_3 j_3\right]\left(\frac{d^2H}{dRdQ}\right)^2 + \frac{(1-v)}{2\beta^2}[q_3 j_2]\left(\frac{d^2H}{dQ^2}\right)^2 + \frac{(1-v)}{2}(q_4 j_2)\left(\frac{dH}{dR}\right)^2\Bigg] dRdQ = 0 \qquad (2.9)$$

$$\frac{d\Pi}{dj_3} = \frac{D}{a^4}\int_0^1\int_0^1 \frac{1}{\beta^2}[-q_2 j_1]\left(\frac{d^2H}{dRdQ}\right)^2 + \frac{(1-v)}{\beta^2}\left[\frac{1}{2}q_3 j_2\right]\left(\frac{d^2H}{dRdQ}\right)^2$$

$$+\frac{(1-v)}{2\beta^2}[q_3 j_3]\left(\frac{d^2H}{dQ^2}\right)^2 + \frac{(1-v)\rho^2}{2\beta^2}(q_4 j_3)\left(\frac{dH}{dQ}\right)^2\Bigg] dRdQ = 0$$

$$-\frac{abNx{j_1}^2}{2}\int_0^1\int_0^1 \left(\frac{\partial w}{\partial x}\right)^2 dRdQ \qquad (2.10a)$$

A parameter λ_{ij} is introduced in Equation (2.10b), deducing their values from Equations (2.8) to (2.10a), Equation (2.10b) is obtained.

$$\lambda_{11} = q_1\left(k_1 + 2\frac{k_2}{\beta^2} + \frac{k_3}{\beta^4}\right), \lambda_{12} = -q_2\left(k_1 + \frac{k_2}{\beta^2}\right), \lambda_{13} = -q_2\left(\frac{k_2}{\beta^2} + \frac{k_3}{\beta^4}\right)$$

$$\lambda_{22} = q_3 k_1 + \frac{(1-v)}{2\beta^2} q_3 k_2 + \frac{(1-v)\alpha^2}{2} q_4 k_4,$$

$$\lambda_{23} = \frac{(1+v)}{2\beta^2} q_3 k_2,\ \lambda_{33} = \frac{(1-v)}{2\beta^2} q_3 k_2 + \frac{1}{\beta^4} q_3 k_3 + \frac{(1-v)\alpha^2}{2\beta^2} q_4 k_5 \qquad (2.10b)$$

The stiffness coefficient k_i expressions were also deduced from Equation (2.8) to (2.10a). They are obtained here as Equation (2.10c).

$$k_1 = \int_0^1\int_0^1 \left(\frac{d^2h}{dR^2}\right)^2 dRdQ, \qquad k_2 = \int_0^1\int_0^1 \left(\frac{d^2h}{dRdQ}\right)^2 dRdQ k_3 = \int_0^1\int_0^1 \left(\frac{d^2h}{dQ^2}\right)^2 dRdQ$$

$$k_4 = \int_0^1\int_0^1 \left(\frac{dh}{dR}\right)^2 dRdQ, k_5 = \int_0^1\int_0^1 \left(\frac{dh}{dQ}\right)^2 dRdQ, k_6 = -\frac{ab}{2}\int_0^1\int_0^1 \frac{Nx{j_1}^2}{a^2} dRdQ \qquad (2.10c)$$

Defining the q_i values in Equations (2.8) to (2.10a) Equation (2.10d) is obtained.

$$\bar{A} = \frac{h^3}{12}, q_1 = \frac{\left(\int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 dz\right)}{\bar{A}} = 1, q_2 = \frac{\left(\int_{-\frac{h}{2}}^{\frac{h}{2}} zF(z)dz\right)}{\bar{A}}, q_3 = \frac{\left(\int_{-\frac{h}{2}}^{\frac{h}{2}} F(z)^2 dz\right)}{\bar{A}},$$

$$\rho^2 q_{\ 4} = \frac{\left(\int_{-\frac{h}{2}}^{\frac{h}{2}}\left[\frac{dF(z)}{dz}\right]^2 dz\right)}{\bar{A}} \quad (2.10d)$$

Deducing Equation (2.10b) to matrix form, Equation (2.11) is obtained.

$$\begin{bmatrix}\lambda_{11} & \lambda_{12} & \lambda_{13}\\ \lambda_{21} & \lambda_{22} & \lambda_{23}\\ \lambda_{31} & \lambda_{32} & \lambda_{33}\end{bmatrix}\begin{bmatrix}j_1\\ j_2\\ j_3\end{bmatrix} = \frac{a^4}{D}\begin{bmatrix}N_x \frac{}{a^2} k_4\\ 0\\ 0\end{bmatrix}\begin{bmatrix}j_1\\ j_2\\ j_3\end{bmatrix} \quad (2.11)$$

Let $\Gamma_{ij} = \lambda_{ij} \times \frac{1}{k_4}$(2.12)

Dividing through Equation (2.11) by Equation (2.12) Equation (2.13) is obtained

$$\begin{bmatrix}\Gamma_{11} & \Gamma_{12} & \Gamma_{13}\\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23}\\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33}\end{bmatrix}\begin{bmatrix}j_1\\ j_2\\ j_3\end{bmatrix} = \frac{a^2}{D}\begin{bmatrix}N_x \frac{}{a^2}\\ 0\\ 0\end{bmatrix}\begin{bmatrix}j_1\\ j_2\\ j_3\end{bmatrix} \quad (2.13)$$

Determining the determinant of the matrix in Equation (2.13), Equation (2.14) is obtained.

$$\Gamma_{11}j_1 + \Gamma_{12}j_1\left[\frac{-\Gamma_{23}\Gamma_{31}+\Gamma_{33}\Gamma_{21}}{\Gamma_{23}\Gamma_{32}-\Gamma_{33}\Gamma_{22}}\right] + \Gamma_{13}j_1\left[\frac{-\Gamma_{23}\Gamma_{21}+\Gamma_{22}\Gamma_{31}}{\Gamma_{32}\Gamma_{23}-\Gamma_{33}\Gamma_{22}}\right] = \frac{j_1 a^2 N_{xcr}}{D} \quad (2.14)$$

For nontrivial solutions, the determinant of the coefficient matrix in Equation (2.14) must be zero. This also presents expression (2.15) as the non-dimensional critical buckling load parameter of the thick plate:

$$\Gamma_{11} + \Gamma_{12}\left[\frac{-\Gamma_{23}\Gamma_{31}+\Gamma_{33}\Gamma_{21}}{\Gamma_{23}\Gamma_{32}-\Gamma_{33}\Gamma_{22}}\right] + \Gamma_{13}\left[\frac{-\Gamma_{23}\Gamma_{21}+\Gamma_{22}\Gamma_{31}}{\Gamma_{32}\Gamma_{23}-\Gamma_{33}\Gamma_{22}}\right] = \frac{a^2 N_{xcr}}{D} = \Phi_a \quad (2.15)$$

Substituting $\rho = a/h$ and the D value in to the right side of Equation (2.14) and making N_{xcr} the subject of the formula, Equation (2.16) is obtained.

$$N_{xcr} = \phi_a * \frac{\mathrm{Eh}}{12(1-v^2)} * \frac{h^2}{a^2} \quad (2.16)$$

$$D = \frac{Eh^3}{12(1-v)} \quad (2.17)$$

$$\frac{b^2 N_{xcr}}{D\beta^2} = \Phi_a \quad (2.18)$$

$$\frac{b^2 N_{xcr}}{D} = \beta^2\Phi_a = \Phi_b \quad (2.19)$$

With Further simplifications, Equations (2.20) and (2.21) are obtained

$$\psi_a = \phi_a * \left(\frac{1}{\rho^{\ 2}}\right) = \frac{\phi_a}{\rho^{\ 2}} \quad (2.20)$$

$$\psi_b = \phi_b * \left(\frac{1}{\rho^{\ 2}}\right) = \frac{\phi_b}{\rho^{\ 2}} = \frac{\phi_a \beta^2}{\rho^2} \quad (2.21)$$

Where:

D is the flexural rigidity of the plate.

N_{xcr} is the critical buckling in-plane load applied to the plates under study.

ϕ_a is the non-dimensional critical buckling load parameter of the plate in the x-direction and.

ϕ_b the non-dimensional critical buckling load parameter of the plate in the y-direction.

Formulation of Polynomial Shear Deformation Function

Polynomial shear deformation function $f(z)$ is a function that describes the deformed shape of the normal to the mid-plane of the plate after deformation has taken place. The function was determined and is presented here as Equation (2.22)

$$f(z) = \frac{z}{5}\left[\frac{99}{20} - 7\left(\frac{z}{h}\right)^2\right] \quad (2.22)$$

$S = \frac{Z}{h}$ or $z = hz$ (2.23)

Where:

'S' is a non-dimensional parameter along the z-axis, and 'h' the thickness of the plate

Rectangular Thick Plate with all Edges Simply Supported (SSSS)

The general polynomial displacement function $w = (x, y)$ as adopted for this research are given in Equation (2.24) and (2.25).

$$w_x = a_0 + a_1 R + a_2 R^2 + a_3 R^3 + a_4 R^4 + a_5 R^5 \quad (2.24)$$

$$w_y = a_0 + a_1 Q + a_2 Q^2 + a_3 Q^3 + a_4 Q^4 + a_5 Q^5 \quad (2.25)$$

Where,

w_x is the out of plane displacement along the x-axis and

w_y is the out of plane displacement along the y-axis

$$w = w_x . w_y = (a_0 + a_1 R + a_2 R^2 + a_3 R^3 + a_4 R^4 + a_5 R^5)(a_0 + a_1 Q + a_2 Q^2 + a_3 Q^3 + a_4 Q^4 + a_5 Q^5) (2.26)$$

The polynomial displacement function for SSSS plate was determined and presented here as Equation (2.27)

$$w = (w_x, w_y) = a_4 b_4 (R - 2R^3 + R^4)(Q - 2Q^3 + Q^4) \quad (2.27)$$

Let, $T = a_4 . b_4$ (2.28)

And $H = (R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$, (2.29)

Where:

$a_4 . b_4$ is the Amplitude and

H = Shape Function for the SSSS thick rectangular plate.

Determination of the stiffness coefficient values for SSSS thick plate

The stiffness coefficient k_i values were determined and summarized here in Equation (2.30).

$k_1 = 0.23619$, $k_2 = 0.235918$, $k_3 = 0.23619$, $k_4 = 0.0239$, $k_5 = 0.0239$,

$k_6 = 0.0024213$. (2.30)

B. Numerical Example

An Example Illustrating the Buckling Analysis of SSSS Rectangular Plates

Consider a plate that is simply supported at its four edges (SSSS). Let's assume that the aspect ratio, which is the ratio of the plate's width (b) to its length (a) ($\beta = b/a$) is equal to 1. The span-depth ratio, ($\rho = a/h$) which is the ratio of the plate's length (a) to its thickness (h), is equal to 5. The Poisson's ratio, denoted by *v*', is equal to 0.3. Theq_i valueswere determined and presented here as Equation (3.1)

$q_1 = 1, \quad q_2 = 0.78, \quad q_3 = 0.6156, \; q_4 = 6.0912$ (3.1)

The stiffness coefficient values $'k_i'$in Equation (2.30) is reproduced here as Equation (3.2)

$k_1 = 0.23619, k_2 = 0.235918, k_3 = 0.23619, k_4 = 0.0239, k_5 = 0.0239,$

$k_6 = 0.0024213$ (3.2)

The values of λ_{ij} presented here as Equation (3.3), were determined from the matrix in Equation (2.11).

λ_{11} =0.944216, $\lambda_{12} = -0.36824, \lambda_{13} = -0.36824, \lambda_{22} = 1.470052, \lambda_{23} = 0.0944$

$\lambda_{33} = 1.470052$(3.3)

Equation (3.3) is substituted into Equation (2.12), and the resulting Γ_{ij} values are substituted into Equation (2.15) to yield Equation (3.4).

$\frac{a^2 N_{xcr}}{D} = \phi_a$ =32.2535 (3.4)

Substituting Equations (3.4) in to Equation (2.19),), Equation (3.5) is obtained.

$\Phi_b = \frac{b^2 N_{xcr}}{D} = \beta^2 \Phi_a = 32.2535 * 1^2 = 32.2535$ (3.5)

Substituting Equation (3.5) into Equation (2.21), Equation (3.6) is obtained.

ψ_b =32.2535$*\frac{1^2}{5^2}$ =1.2901 (3.6)

III. RESULTS AND DISCUSSION

A. Results Presentation

The non-dimensional critical buckling load parameter ϕ_a and $\Phi_b$of thick rectangular SSSS plate were obtained and summarised here as Equation (4.1)

ϕ_a =32.25354258, $\Phi_b = 32.25354258,$ (4.1)

i. Buckling Analysis of SSSS thick plates

Table 4.7 is comparing the present study with the research works of other researchers.

Table 4.7: Results of the Present Study Compared with the Results of other Researchers for SSSS thick Plate.

		$\beta = b/a$						
		$N_{xcr} = \phi_a/12(1-v^2)$						
$\rho = a/h$	Theory	1.0	1.5	2.0	2.5	3.0	3.5	4.0
5	Present Study (P.S)	2.9536	1.6233	1.2391	1.0767	0.9928	0.9436	0.9123
	Ezehet *al.* (2018), (E)	2.9556	1.6255	1.2408	1.0783	0.9936	0.9455	0.9135
	Sayyad and Ghugal (2012), (S)	3.0260	1.6540	1.2590	1.0930	1.0070	0.9570	0.9250
	% Difference btw P.S and E	-0.0670	-0.1356	-0.1377	-0.1452	-0.0826	-0.1984	-0.1295
	% Difference btw P.S and S	-2.3919	-1.8563	-1.5813	-1.4882	-1.4122	-1.3977	-1.3712
10	Present Study (P.S)	3.4249	1.8134	1.3654	1.1789	1.0831	1.0273	0.9918
	Ezehet *al.* (2018), (E)	3.4249	1.8155	1.3668	1.1790	1.0852	1.0279	0.9936
	Sayyad and Ghugal (2012), (S)	3.4540	1.8250	1.3730	1.1850	1.0890	1.0320	0.9970
	% Difference btw P.S and E	0.0001	-0.1141	-0.1020	-0.0116	-0.1930	-0.0610	-0.1808
	% Difference btw P.S and S	-0.8421	-0.6340	-0.5531	-0.5178	-0.5412	-0.4581	-0.5212
20	Present Study (P.S)	3.5676	1.8682	1.4012	1.2075	1.1084	1.0506	1.0139
	Ezehet *al.* (2018), (E)	3.5691	1.8704	1.4034	1.2088	1.1103	1.0508	1.0142
	Sayyad and Ghugal (2012), (S)	3.5820	1.8740	1.4050	1.2110	1.1110	1.0530	1.0160
	% Difference btw P.S and E	-0.0425	-0.1160	-0.1589	-0.1044	-0.1755	-0.0203	-0.0279
	% Difference btw P.S and S	-0.4025	-0.3079	-0.2726	-0.2858	-0.2384	-0.2292	-0.2050
50	Present Study (P.S)	3.6097	1.8842	1.4115	1.2158	1.1156	1.0573	1.0203
	Ezehet *al.* (2018), (E)	3.6103	1.8864	1.4125	1.2179	1.1172	1.0577	1.0211
	Sayyad and Ghugal (2012), (S)	3.6210	1.8890	1.4150	1.2190	1.1180	1.0590	1.0220
	% Difference btw P.S and E	-0.0163	-0.1176	-0.0689	-0.1706	-0.1401	-0.0371	-0.0794
	% Difference btw P.S and S	-0.3117	-0.2551	-0.0689	-0.2607	-0.2116	-0.1599	-0.0794
100	Present Study (P.S)	3.6158	1.8865	1.4130	1.2170	1.1167	1.0582	1.0212
	Ezehet *al.* (2018), (E)	3.6172	1.8887	1.4148	1.2179	1.1172	1.0600	1.0234
	Sayyad and Ghugal (2012),(S)	3.6250	1.8910	1.4160	1.2190	1.1190	1.0600	1.0230
	% Difference btw P.S and E	-0.0383	-0.1174	-0.1259	-0.0727	-0.0463	-0.1628	-0.2144
	% Difference btw P.S and S	-0.2534	-0.2389	-0.2106	-0.1628	-0.2071	-0.1628	-0.1754

B. Discussions regarding the results of Buckling Analysis of a Thick Isotropic Rectangular Plate

Table 4.7 presents a comparison of the buckling analysis results of this study with those of other researchers. The percentage difference between the critical buckling loads of this study and that of other researchers is negligible, with a maximum value of 2.3919% at an aspect ratio of β=b⁄a= 1 and a span depth ratio of (ρ=a⁄h) = 5, corresponding To the research conducted by Sayyad and Ghugal (2012). The minimum percentage difference is 0.0001%, corresponding to the research of Ezeh et al. (2018) at an aspect ratio of β=b⁄a = 1 and a span depth ratio of ρ=a⁄h= 10. It is important to note that any result within the range of 0% to 5% is considered acceptable in engineering calculations, based on statistical principles for result comparison. Therefore, this study provides a reliable solution for the buckling analysis of isotropic thick rectangular plates.

IV. CONCLUSIONS AND RECOMMENDATIONS

A Conclusion

Based on the findings of this study, we can conclude that the simultaneous equations utilized in this research provide accurate and satisfactory results for buckling analysis of isotropic thick rectangular plates. The general Orthogonal Polynomial Displacement Functions 'w' used for the analysis of the isotropic rectangular thick SSSS plate are easily adaptable for plates with different boundary conditions, without the need for complicated mathematical equations. In addition, the third order polynomial shear deformation function F(z) developed for this study produced acceptable results that fall within the acceptable statistical interval when compared with other research works, as presented in Table 4.7. This function offers a parabolic variation of the transverse shear strains and stresses across the plate thickness, which eliminates the need for a shear correction factor as used in the first order shear deformation theory (FSDT). Lastly, the stiffness coefficient values ($k_i$values) of the rectangular thick plate with SSSS boundary conditions determined in this research are highly reliable and consistent with the works of Sayyad, Ghugal (2012) and many other researchers.

B. Recommendations

i) This research employs the user-friendly Shear Deformation Function F(z) for thick plate analysis. Designers are encouraged to utilize this function, as its results are both reliable and efficient.
ii) The Orthogonal Polynomial Displacement Functions developed for this study prove highly effective for analyzing thick rectangular SSSS plates. It is suggested that this efficiency could extend to other complex plate systems with multiple unsupported edges.
iii) Future researchers should consider applying the theories presented in this study to explore the combined effects of in-plane compression, transverse, and dynamic loads on thick rectangular plates.
iv) To achieve more efficient results for critical buckling load of thick plates, it is recommended that further research be conducted on the Shear Deformation Theory of fifth and seventh order.

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