

Modified Horizontal Stress Equations for Rock Mass Insitu Stress State

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ABSTRACT : *The in-situ stresses of a rock mass are very important parameters in engineering rock mechanics. Currently in available literature there is only one classical equation derived by Dinnik in 1925 based on elasticity theory. This equation is a function of only the Poisson ratio. The aim of this work is to provide alternative equations for predicting the horizontal component of the insitu stress state of a rock mass with ease especially where the Poisson ratio cannot be determine in the field, which will make it difficult to apply the existing Dinnik equation. The procedure adopted here involves adjusting mathematically the existing equation to get Poisson ratio and substituting for the Poisson ratio in the Dinnik equation to obtain the new four modified alternative equations for predicting the rock mass insitu horizontal stress state component. These new equation will provide more ease of getting this result for engineering use.*

Keywords: In-situ stress, Horizontal stress state, Rock mass, Elastic constant, Equations

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I. INTRODUCTION

The International Association of Engineering Geology Statutes, 1992 as cited by Peng and Zhang (2007), states that Engineering Geology is focused on the investigation, study and solution of the engineering and environmental problems which may arise as the result of the interaction between geology and the works and activities of man as well as to the prediction and the development of measures for prevention or remediation of geological hazards. Rock mechanics a major aspect of geological and geotechnical engineering is a field of applied science which consists of a body of knowledge of the mechanical properties of rock, various techniques for the analysis of rock stress under some imposed perturbation, a set of established principles expressing rock mass response to load, and a logical scheme for applying these notions and techniques to real physical problems (Brady and Brown, 2005). They further states that, rock mass is the total in-situ medium

containing bedding planes, faults, joints, folds and other structural feature. This rock mass is discontinuous and heterogeneous in nature with anisotropic engineering properties.

Rocks are subjected to stresses. The determination of in-situ stresses of a rock mass is very important in engineering rock mechanics. According to Hayavi and Abdideh (2016), accurate estimation of in situ stresses of a subsurface formation is important to get a basic knowledge of formation structure and position of anomalies, groundwater flows, performing fracturing operations etc. There is one classical equation derived by Dinnik in 1925 based on elasticity theory as reported by Turchaninor et al (1979) and Hudson and Harrison (1997). This equation presents the horizontal in-situ state stress as directly proportional to the vertical stress with the involvement of the Poisson ratio, which is the material property. This implies that the onsite measurement of the Poisson ratio need to be

carried out before this equation can be used. However, it is common knowledge that some terrain are not accessible for direct measurement of rock deformations leading to Poisson ration estimation. This implies that in such terrain it will be difficult to apply this equation. However, other elastic properties such as young modulus, bulk modules and shear modulus can be obtained easily using geophysical surveys and simple empirical relations. Geophysical surveys are useful and powerful indirect methods for engineering site investigation and geological studies of rocks. Most scholars have applied this both engineering and geological studies (Kovačević et al., 2013; Coker et al.,2013; Coker,2015).

The aim is to formulate or provide alternative equations for predicting the horizontal component of the insitu stress state of a rock mass with ease especially where the Poisson ratio cannot be determined in the field, which will make it difficult to use the existing Dinnik equation. Geophysical methods can be used to obtain rock elastic parameter easily which can be used with the new equations to predict the horizontal in-situ stress state of a rockmass with ease.

II. METHODOLOGY

A. FORMULATION OF EQUATIONS

Dinnik in 1925 as reported by Turchaninor et al (1979) and Hudson and Harrison (1997) formulated the horizontal component of the rock mass insitu stress state, σ_H , based on elasticity theory as given by Equation 1.

$$\sigma_H = \frac{\nu}{1 - \nu} \sigma_V \quad (1)$$

Where, ν is Poisson ratio and σ_V is the vertical stress component of the rock insitu stress state given by

$$\sigma_V = \gamma Z = \rho g Z \quad (2)$$

Where γ is unit weight measured in MN/m³ and Z is the depth in meters, ρ is density and g is acceleration due to gravity.

From theory of elasticity, the shear modulus, μ , is given as Equation (3) (Ibearugbulem, et al., 2014; Khurmi and Khurmi, 2013; Shendi, 2007).

$$\mu = \frac{E}{2(1 + \nu)} \quad (3)$$

Equation (3) can be expressed as

$$\mu = \frac{E(1 - \nu)}{2(1 - \nu^2)} \quad (4)$$

Rearranging yields

$$1 - \nu = \frac{2\mu(1 - \nu^2)}{E} \quad (5)$$

Substituting Equation (5) into equation (1) yields

$$\sigma_H = \frac{\nu E}{2\mu(1 - \nu^2)} \sigma_V \quad (6)$$

Also, the bulk modulus, K , is given by

$$K = \frac{E}{3(1 - 2\nu)} \quad (7)$$

Rearranging yields

$$E = 3K(1 - 2\nu) \quad (8)$$

Substituting equation (8) into equation (6) yields

$$\sigma_H = \frac{3\nu K(1 - 2\nu)}{2\mu(1 - \nu^2)} \sigma_V \quad (9)$$

From equation (3)

$$\nu = \frac{E}{2\mu} - 1 \quad (10)$$

Substituting Equation (10) into equation (1) yields

$$\sigma_H = \frac{\frac{E}{2\mu} - 1}{2 - \frac{E}{2\mu}} \sigma_V \quad (11)$$

Simplifying yields

$$\sigma_H = \frac{E - 2\mu}{4\mu - E} \sigma_V \quad (12)$$

From equation (8)

$$\nu = \frac{1}{2} - \frac{E}{6K} \quad (13)$$

Substituting equation (13) into Equation (1) yields

$$\sigma_H = \frac{\frac{1}{2} - \frac{E}{6K}}{1 - \frac{1}{2} + \frac{E}{6K}} \sigma_V$$

Simplifying yields

$$\sigma_H = \frac{3K - E}{3K + E} \sigma_V \quad (14)$$

Equations (6), (9), (12) and (14) are the new modified horizontal component insitu rock stress state.

From equations (1), (6), (9), (12) and (14)

$$\frac{\nu}{1 - \nu} = \frac{\nu E}{2\mu(1 - \nu^2)} = \frac{3\nu K(1 - 2\nu)}{2\mu(1 - \nu^2)} = \frac{E - 2\mu}{4\mu - E} = \frac{3K - E}{3K + E} \quad (15)$$

Equations (6), (9), (12) and (14) are the four new modified equations for predicting the horizontal component of the insitu rock stress state for geological and engineering works purposes.

From equation (1) and Equation (9)

$$\mu = \frac{3K(1 - 2\nu)}{2(1 + \nu)} \quad (16)$$

Equating equation (12) and Equation (14) yields

$$\frac{E - 2\mu}{4\mu - E} = \frac{3K - E}{3K + E}$$

Simplifying yields

$$\mu = \frac{3EK}{9K - E} \quad (17)$$

Equating equations (17) and (16) yields

$$\frac{3EK}{9K - E} = \frac{3K(1 - 2\nu)}{2(1 + \nu)}$$

Simplify

$$9EK - 27K^2 = -54K^2\nu$$

From which

$$\nu = \frac{27K^2 - 9EK}{54K^2}$$

Resolving further yields

$$\nu = \frac{1}{2} - \frac{E}{6K} \quad (18)$$

Equation (18) and (13) are the same confirming that the equations are correct.

III. RESULTS AND DISCUSSION

The new modified equations from this work are presented in Table 1.

Table 1: Modified new Equations for Insitu horizontal Stress State, σ_H

| Parameter | Modified New Equations |
|--|--|
| Insitu horizontal Stress State, σ_H | $\sigma_H = \frac{\nu E}{2\mu(1 - \nu^2)} \sigma_v$ |
| | $\sigma_H = \frac{3\nu K(1 - 2\nu)}{2\mu(1 - \nu^2)} \sigma_v$ |
| | $\sigma_H = \frac{E - 2\mu}{4\mu - E} \sigma_v$ |
| | $\sigma_H = \frac{3K - E}{3K + E} \sigma_v$ |

This equations provide alternatives means of predicting the insitu horizontal Stress State, σ_H , based on the given data available instead of just Poisson ratio only as is found in literature. This becomes even more important in situation where it is not possible to carry out direct measurement of lateral and axial strain in the field to compute Poisson ratio, Seismic geophysical data can be collected and the other elastic parameters computed, and the horizontal stress state can be determine easily using the applicable equation from this work.

From equation (18),

$$\text{If } \nu = 0; K = \frac{E}{3} \quad (19)$$

Compare this result of equation (19) with the one in literature from equation (7)

$$K = \frac{E}{3(1 - 2 * 0)} = \frac{E}{3} \quad (20)$$

This shows that the new approach is adequate.

Substituting for K from equation (19) or (20) into Equation (16) or (17) yields

$$\mu = \frac{E}{2} \quad (21)$$

Substituting equation (19) into Equation (21) yields

$$\mu = \frac{3K}{2} \quad (22)$$

Based on equations (19) and (20), the insitu horizontal stress state component, σ_H , from the new equations (6), (9), (12) and (14) will be zero. This is in conformity with equation (1) when the Poisson ratio, ν is zero (Hudson and Harrison, 1997). From equation (22), when the Poisson

ration is zero, the shear modulus is directly proportional to the bulk modulus.

III. CONCLUSION AND RECOMMENDATIONS

The present study has formulated alternative equations in terms of stiffness parameters other than Poisson ratio for the determination of the horizontal component of the rock mass insitu stress state. These equations are useful alternatives to calculate the horizontal stress state of a rock mass especially where direct measurement of on-site Poisson ratio is humanly impracticable due to terrain. A comparison of these new equation with the existing one in literatures indicate that the new equation are very adequate for obtaining the horizontal stress state of a an in-situ rock mass. These equations will be of help to engineering geologists, structural geologists and geotechnical engineers to accomplish their work with ease.

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