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Optimization Model of Biogas Production efficiency from different sources of Waste by Anaerobic digestion

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Abstract

This research paper focuses on the optimization modeling of biogas production efficiency from various sources of waste using anaerobic digestion. The study was conducted in a controlled laboratory environment employing mini-biodigesters with a capacity of 120 liters. Four different feedstocks—sewage, pig waste, poultry waste, and homemade waste (a mixture of watermelon and pineapple)—were used as substrates to feed the biodigesters individually and in various combinations. The experimental results over a 14-day maximum production period indicated average daily biogas production rates of 0.0329, 0.0372, 0.0354, 0.0296, 0.0362, 0.0384, and 0.0410 liters per day for the substrates: sewage waste (X1), pig waste (X2), poultry waste (X3), homemade waste (X4), the combination of sewage and homemade waste (X1+X4), the combination of sewage, pig, and poultry waste (X1+X2+X3), and the combination of all four substrates (X1+X2+X3+X4), respectively. The methane content in the biogas produced from these substrates was found to be 54.8%, 58.7%, 56.6%, 51.7%, 68.2%, 65.5%, and 69.3%, respectively. To optimize biogas production, a mathematical model was formulated with the variables X1, X2, X3, and X4 subjected to time constraints. The Simplex Method was employed to solve the model, resulting in an objective function value of 12.22286. The optimal values of the variables were X1 = 0, X2 = 4.285714, X3 = 1.428571, and X4 = 1.78E-16, indicating the most efficient combination of feedstocks for biogas production. This study provides significant insights into the efficiency of biogas production from different waste sources and their combinations, contributing to the optimization of anaerobic digestion processes for sustainable energy production.

Keywords: *Agricultural wastes; Biogas production; Homemade wastes; Optimization Model; Sewage wastes Digestion.*

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1. INTRODUCTION

As global energy demands continue to rise and environmental concerns intensify, there is an urgent need to explore sustainable and renewable energy sources. Biogas, produced through the anaerobic digestion of organic waste, presents a promising solution. Not only does biogas serve as a renewable energy source, but it also offers a practical method for waste management, reducing environmental pollution and contributing to the circular economy.

Anaerobic digestion is a biochemical process where microorganisms break down organic material in the absence of oxygen, producing biogas primarily composed of methane (CH4) and carbon dioxide (CO2). The efficiency of biogas production can vary significantly depending on the type of organic waste used as feedstock. Understanding the potential of different waste materials, both individually and in combination, is crucial for optimizing biogas yield and methane content.

This research explores the production of biogas from various organic waste feedstocks using two 120-liter mini-biodigesters in a controlled laboratory environment. The study investigates the biogas yield and methane content produced from four different feedstocks: sewage waste, pig waste, poultry waste, and homemade waste composed of watermelon and pineapple residues.

These feedstocks were tested individually and in combinations to evaluate their efficiency in generating biogas and to identify optimal mixtures for improved production.
Key Findings:
Biogas Yield: The average daily biogas production over the 14-day peak production period varied across the substrates, with the highest output of 0.0410 liters observed when all four feedstocks were combined. Individually, pig waste produced the highest rate of 0.0372 liters, followed by poultry waste (0.0354 liters) and sewage waste (0.0329 liters).
Methane Content: The methane concentration in the biogas also showed variation. The combination of all four feedstocks resulted in the highest methane content (69.3%), while individual feedstocks like pig waste (58.7%) and poultry waste (56.6%) produced moderately high levels of methane. Homemade waste had the lowest methane content at 51.7%.
Combination Benefits: The study revealed that combining multiple feedstocks, particularly sewage waste, pig waste, poultry waste, and homemade waste, significantly enhanced both the biogas yield and methane content. The mixture of sewage and homemade waste achieved a notable methane content of 68.2%.

Biogas and Anaerobic digestion

Biogas is a renewable, high-quality fuel, which was produced from a lot of different organic raw materials and be used for various energy services. Biogas technology has been developed and widely used around the world, because it has a lot of advantages, including reducing the dependence on non-renewable resources; high energy efficiency; preventing environmental pollution; available and cheap resources to feedstock; relatively easy and cheap technology for production; and extra values of digestate as fertilizer, etc. But the status of biogas production and utilization largely varies among the different continents. Biogas is produced when microorganisms degrade organic materials in the absence of oxygen [1].
Biogas was used as fuel for heating, cooking, and electricity generation. It was burned directly in boilers and engines or upgraded to natural gas quality and injected into the gas grid. Biogas is considered a renewable energy source because the organic matter used to produce biogas was replenished through agricultural and food waste streams. The production of biogas has several environmental benefits, such as; reducing greenhouse gas emissions, providing a sustainable waste management solution, and contributing to energy independence. However, the production of biogas also requires careful management to ensure proper operation of the anaerobic digestion process and to prevent environmental impacts such as odor emissions and nutrient runoff [1].
This process of producing Biogas is known as anaerobic digestion and it produces a gas mixture that is composed primarily of methane (CH4) and carbon dioxide (CO2), with small amounts of other gases such as hydrogen sulfide (H2S) and ammonia (NH3). The feedstock can derive from agricultural, industrial, or municipal sources. To date, to obtain a higher biogas yield, a lot of agricultural biogas plants digest manure with some additional co-substrates for increasing the content of organic materials. Besides input materials, biogas yield and anaerobic processes are affected by several other factors. There are a lot of different types of biogas plants all over the world, and they are accepted and widely used by different countries. For example, floating drums and fixed dome biogas plants are two major types of small to medium-scale biogas digesters used in African countries.
The microscopic organism that produces biogas is known as Archaea. Archaea are among the oldest life forms on Earth. They are much less oxygen-breathing and CO2-absorbing plant life, that preexisted on planet Earth 3.5 billion years ago. They are not bacteria but are genetically closer to humans and other animals (eukaryotes) and form their own animal kingdom. As the earth's atmosphere became predominantly oxygen about 500 million years ago, Archaea became isolated in the few remaining airless places, such as stagnant swamps, deep oceans, caves, hot springs, and of course the stomachs of vertebrates. To create biogas, we must recreate the conditions in which Archaea thrive in nature. Biogas is reproduced in a special air-tight tank called an anaerobic digester [1].
Natural biodegradation of organic matter contributes approximately 590-800 million tons of methane to the atmosphere [2]. Wastewater and landfills constitute 90% of waste sector emissions and about 18% of global anthropogenic methane (CH4) emissions [3]. Methane (CH4) which has a high potential for global warming can either be tapped or released freely into the atmosphere. The latter situation takes place when organic matters are illegally disposed of or thrown away in vacant places. The tapped methane (Biogas) is used as a source of energy, while the untapped methane is very harmful to the environment [2]
Anaerobic digestion is the process and technique of decomposition of organic matter by a microbial process in an oxygen-free environment [4]. Controlled anaerobic digestion of organic waste has multiple benefits. On the one hand, it provides a renewable source of clean energy, while on the other side, the digestates was used

as organic fertilizers in the agriculture sector. The electricity and fuel production from the biogas might strengthen the national energy supply, as well as reduce greenhouse gas (GHG) emissions [5].

Reason for Adopting the Simplex Method of Model Analysis over Non-linear dynamics and interaction effects for this work:
Non-linear dynamics and interaction effects are often complex and difficult to model in systems with multiple interacting variables, such as biogas production from different substrates. These approaches account for unpredictable or chaotic behaviors that may arise from the interactions between different feedstocks and operational parameters like temperature, pH, or microbial activity. While non-linear models are powerful in capturing such complexities, they require sophisticated analysis, which may introduce additional uncertainties or computational challenges in a lab-scale study like this one.
The simplex method, on the other hand, is a linear optimization technique that simplifies the modeling process. It focuses on finding the optimal combination of feedstocks for maximum biogas production and methane content, without needing to model the complex, often unpredictable interaction effects between variables. The simplex method is used here because it provides a straightforward, linear approach to optimize biogas yield from different feedstock combinations, offering clear and interpretable results suitable for this controlled laboratory experiment.
In essence, the choice of the simplex method over non-linear dynamics ensures a simpler, more manageable model that is more appropriate for the scale and scope of this study, focusing on optimization rather than capturing the complexities of interaction effects.
Reason for Adopting the Simplex Method of Model Analysis over other optimization techniques like Genetic Algorithms, Artificial Neural Networks for this work:
The Simplex Method is chosen over other optimization techniques like Genetic Algorithms (GA) or Artificial Neural Networks (ANN) for several reasons, particularly because of its suitability for the nature and goals of this study on biogas production:

1. Simplicity and Interpretability
The Simplex Method is a linear programming technique that is relatively simple to implement and understand. It works well when the relationships between variables are assumed to be linear, which is the case in this study where the goal is to optimize biogas yield based on straightforward combinations of feedstocks. Its results are clear and interpretable, making it easier for researchers to directly see how different feedstock combinations affect the outcome.
In contrast, Genetic Algorithms and Artificial Neural Networks are more complex and involve non-linear modeling. While they can handle more intricate interactions and non-linear relationships, their results can be harder to interpret and require more computational resources, which may not be necessary for a relatively small-scale, controlled laboratory study like this.

2. Data Availability and Size
The Simplex Method is effective for optimization problems with a smaller number of variables and constraints. In this study, the number of variables (different feedstock types and combinations) is limited, and the relationships are well-defined. The method can efficiently solve the problem of maximizing biogas yield with a smaller dataset.
Genetic Algorithms and Artificial Neural Networks are more suited for larger datasets with many variables and complex, non-linear interactions. They require extensive data for training (ANNs especially) to perform well, which might not be available in this experimental setup, where a limited number of trials are conducted.

3. Optimization Goals
The Simplex Method is designed to solve optimization problems where the objective is to maximize or minimize a linear function (such as biogas production or methane content) subject to a set of linear constraints. This aligns well with the study's goals of identifying the optimal combinations of feedstocks for biogas production, where the effects of different combinations can be modeled linearly.
Genetic Algorithms are heuristic methods based on evolution and can be computationally expensive, especially for simple optimization tasks. They are better suited for complex, multi-objective problems where the search space is large and includes non-linear, discontinuous variables.
Artificial Neural Networks are excellent for pattern recognition and predictive modeling in highly complex, non-linear systems but are overkill for the relatively simple optimization problem in this research. Additionally, they require more sophisticated training and validation processes, which may not be necessary or efficient for this study.

4. Computational Efficiency

The Simplex Method is computationally efficient and can quickly converge on an optimal solution, making it ideal for this small-scale, controlled environment. It requires fewer computational resources and less time compared to Genetic Algorithms or Artificial Neural Networks, which may involve iterative processes, random searches, or multi-layered computations that are more time-consuming and resource intensive.

5. Linear Assumptions

The assumptions of linearity in the Simplex Method are appropriate for this study, where the relationships between input variables (feedstock combinations) and outputs (biogas yield and methane content) are expected to be relatively linear or can be approximated as such. This makes the Simplex Method a more straightforward and appropriate choice.

Genetic Algorithms and Artificial Neural Networks are more suited to highly non-linear and complex optimization problems, which is not the primary focus of this study. For a linear optimization problem, the additional complexity of these methods is unnecessary.

Table 1: Summary Review of Past works in Biogas

S/N	PROJECT TITLE	NAME OF AUTHORS	NO. OF FEEDSTOCK	MODELING/ STATISTICAL ANALYSIS	AREA NOT COVERED
1	Comparative Study of Biogas Generation from Chicken Waste, Cow Dung and Pig Waste Using Constructed Plastic Bio Digesters	[6]	3	No	Sewage and Home-made wastes
2	Preparation of Biogas from Plants and Animal Waste	[7]	9	No	Sewage and Home-made wastes
3	Evaluation of Biogas Yield and Microbial Species from Selected Multi-biomass Feedstocks in Nigeria	[8]	6	Yes, Statistical Analysis	Sewage and Home-made wastes
4	Evaluation of Biogas Production from Food Waste	[9]	4	Yes, Statistical Analysis	Sewage and Agricultural wastes
5	Co-digestion of sewage sludge and organic fraction of municipal solid waste	[10]	2	No	Home made and Agricultural wastes
6	**Harvesting biogas from wastewater sludge and food waste**	[11]	2	No	Agricultural Waste
7	Analyses of Anaerobic Batch Digestion of Municipal Solid Waste in the Production of Biogas Using Mathematical Models	[12]	1	Yes, Mathematical Model	Home made and Agricultural wastes
8	Anaerobic digestion models No 1 (ADM1)	[13]	0	Yes, Mathematical Model	Home made, Sewage, and Agricultural wastes
9	Modeling of biogas production by anaerobic digestion for the generation of electricity.	[14]	0	Yes, Mathematical Model	Homemade, Sewage, and Agricultural wastes
10	Optimization of biogas production from multiple feedstocks through co-digestion.	[15]	0	Yes, Mathematical Model	Homemade, Sewage, and Agricultural wastes
11	Biogas production from co-digestion of multiple organic wastes: A review	[16]	Multiple	No	Sewage
12	Synergistic effect of co-digestion on biogas production from mixed organic substrates	[17]	5	No	Sewage
13	Biogas production from co-digestion of multiple organic wastes: An integrated approach	[18]	3	No	Home made, and Sewage
14	Enhanced biogas production through co-digestion of multiple feedstocks: A review	[19]	4	No	Sewage

15	Biogas production from co-digestion of sewage sludge with various organic wastes	[20]	0	No	Sewage
16	Co-digestion of multiple organic wastes for enhanced biogas production: A techno-economic analysis	[21]	4	No	Sewage
17	Biogas production from co-digestion of multiple feedstocks: A comparative study	[22]	4	No	Sewage
18	integration of multiple feedstocks for biogas production: A feasibility study	[23]	3	No	Sewage
19	Biogas production from co-digestion of multiple substrates: A review on process challenges and strategies	[24]	5	No	-
20	Methane Production from Anaerobic Co-digestion of Cow Dung, Chicken Manure, Pig Manure and Sewage Waste	[25]	4	No	Homemade waste

II. METHODS

Materials: Feedstocks (Substrates)

Four types of organic waste materials were selected as substrates for biogas production. Each was chosen based on its availability and potential for biogas generation:

1. Sewage Waste (X1): Collected from a local wastewater treatment plant. The waste was stored in sealed containers to maintain anaerobic conditions prior to use.
2. Pig Waste (X2): Obtained from a local pig farm, consisting of feces and other organic matter. The waste was freshly collected, homogenized, and stored under anaerobic conditions.
3. Poultry Waste (X3): Collected from a poultry farm, primarily consisting of feces, feathers, and leftover feed. The waste was prepared similarly to pig waste.
4. Homemade Waste (X4): Comprised of watermelon and pineapple residues from household kitchen waste. The fruits were blended into a slurry to ensure even consistency before being used as a substrate.

Biodigesters

Two laboratory-scale biodigesters with a capacity of 120 liters each were used in this experiment. Both biodigesters were fabricated from high-density polyethylene (HDPE) material to ensure durability and minimize leakage. The digesters were equipped with:

- Gas Collection System: Each biodigester was connected to a gas storage bag via a biogas flow meter to measure the daily biogas production.
- Mixing Mechanism: Manual stirring was performed twice a day to maintain homogeneity of the substrate and prevent stratification.
- Temperature Control: The biodigesters were housed in a controlled environment to maintain a constant mesophilic temperature (35°C), which is optimal for anaerobic digestion.

Experimental Setup

Biodigester Loading

The feedstocks were added to the biodigesters in two phases:

- Single-Substrate Loading: Each biodigester was initially loaded with one type of substrate (sewage, pig, poultry, or homemade waste). This setup allowed the evaluation of biogas production and methane content from each feedstock individually.
- Combined Substrate Loading: After single-substrate tests, combinations of feedstocks were loaded into the biodigesters to assess whether mixed substrates could enhance biogas yield. The combinations tested were:
    - Sewage Waste + Homemade Waste (X1 + X4)
    - Sewage Waste + Pig Waste + Poultry Waste (X1 + X2 + X3)
    - Sewage Waste + Pig Waste + Poultry Waste + Homemade Waste (X1 + X2 + X3 + X4)

Each biodigester was filled to 80% of its total volume to allow sufficient space for gas production and accumulation.

Hydraulic Retention Time (HRT)

The experiment was conducted over a 30-day period for each feedstock setup, but the results reported focus on the 14 days of maximum production. The retention time was kept constant at 30 days to ensure that the substrates had ample time for digestion.

Monitoring and Data Collection

The following parameters were monitored daily:

1. Biogas Production Rate: Daily biogas production was measured using a gas flow meter attached to each biodigester. The data were recorded in liters per day.
2. Methane Content: The methane concentration in the biogas was analyzed using a gas chromatograph (GC) calibrated to detect methane. Samples of biogas were collected daily and tested to determine the methane percentage.
3. pH Levels: The pH of the substrate inside the biodigesters was monitored every 3 days using a portable pH meter. The optimal pH range for anaerobic digestion (6.5–7.5) was maintained by adjusting with either citric acid or sodium bicarbonate as necessary.
4. Temperature: The temperature inside the biodigesters was kept constant at 35°C using a thermostat-controlled heating element. Temperature fluctuations were recorded using digital temperature sensors installed in each biodigester.

Assumptions in the Model

1. Homogeneity of Feedstocks: It is assumed that each feedstock (sewage, pig, poultry, homemade) is homogeneous in composition. In reality, the composition of organic waste can vary, but the model assumes uniformity in the properties of the feedstocks used.

2. Constant Temperature: The model assumes that the biogas production occurs at a constant temperature, ideally within the mesophilic (25–40°C) or thermophilic (50–60°C) range. This simplifies the analysis of gas production, though in reality, temperature fluctuations could affect biogas yield and methane content.

3. No Inhibitory Compounds: The model assumes that there are no inhibitory substances (such as high ammonia levels, heavy metals, or pH imbalances) that could negatively affect the anaerobic digestion process. The feedstocks are considered to be free of toxic compounds that could hinder microbial activity.

4. Optimal pH Range: The model assumes that the pH levels within the biodigesters remain within the optimal range (6.5–7.5) for biogas production throughout the experiment. Variations in pH could affect microbial efficiency, but this is assumed to be controlled or stable in the model.

5. Stable Microbial Activity: It is assumed that the microbial populations responsible for anaerobic digestion are stable and functioning optimally throughout the 14-day production period. Microbial growth and activity can fluctuate, but the model presumes consistent activity.

6. Ideal Mixing Conditions: The model assumes that there is adequate and uniform mixing of feedstock materials within the biodigester. This ensures that all the biomass is equally exposed to the anaerobic microorganisms, even though mixing efficiency can vary in real scenarios.

7. No Gas Loss: The model assumes that there is no loss of biogas during collection, storage, or measurement. This simplifies the assessment of biogas yield, although in practice, there can be minor losses due to leaks or measurement inaccuracies.

8. Constant Retention Time: The model assumes that the retention time for the feedstock in the biodigester is constant and uniform, ensuring that each substrate has equal exposure to the anaerobic digestion process. Variations in retention time could influence the overall gas production.

9. Uniform Initial Substrate Loading: The model assumes that the quantity of substrate fed into the biodigesters is the same for each experimental run. In practice, slight variations in loading rates can affect gas yield, but the model assumes consistent loading to ensure comparability.

10. Steady-State Conditions: It is assumed that the biodigesters reach a steady state after the initial start-up phase and that gas production occurs consistently during the 14-day peak production period. Start-up and lag phases are neglected for simplicity.

11. No External Interference: The model assumes no external interference, such as changes in pressure, humidity, or unexpected microbial contamination, which could alter biogas production dynamics.

12. Negligible Effect of Byproducts: It is assumed that byproducts such as digestate (solid waste) do not significantly impact the anaerobic digestion process. The focus is solely on gas production, even though byproducts can sometimes affect biogas yields.

13. Methane Content Assumed Proportional to Feedstock: The model assumes that the methane content of the biogas produced is directly proportional to the type and combination of feedstocks used. Variations in microbial performance or digestion efficiency are assumed to be minimal in determining methane output.

III. RESULTS AND DISCUSSION

Sewage Waste, Agricultural wastes (Pig and Poultry wastes) Home-made food waste (Pineapple and watermelon peels).

Table 2: Table showing summary of Findings from the laboratory work for all feedstock Wastes

Parameter	**Percentage (%)**			
	SEWAGE WASTE	**PIG WASTE**	**POULTRY WASTE**	**HOMEMADE WASTE**
Organic Matter	7.8%	20.5%	46.42%	91%
Moisture Content	99%	70%	30.09%	13.5%
Carbon Content	8.64%	58%	36.12%	35.1%
Nitrogen Content	0.32%	5.8%	2.40%	1.3%
Carbon-to-Nitrogen (C/N) Ratio	26:1	10:1	15:1	27:1
Ph	6.7	6.5	6.94	6.3
Temperature	22℃	20℃	20℃	36.4℃

From table 2 above, the PH falls around the neutral 6.30 to 6.94; the Carbon-Nitrogen ratio is between 10:1 to 27:1.

Biogas Production Rates: The biogas production rate refers to the amount of biogas generated per unit of time from an anaerobic digestion process. The biogas production rate for the feedstocks were measured in cubic meters per day as seen in table 3 below.

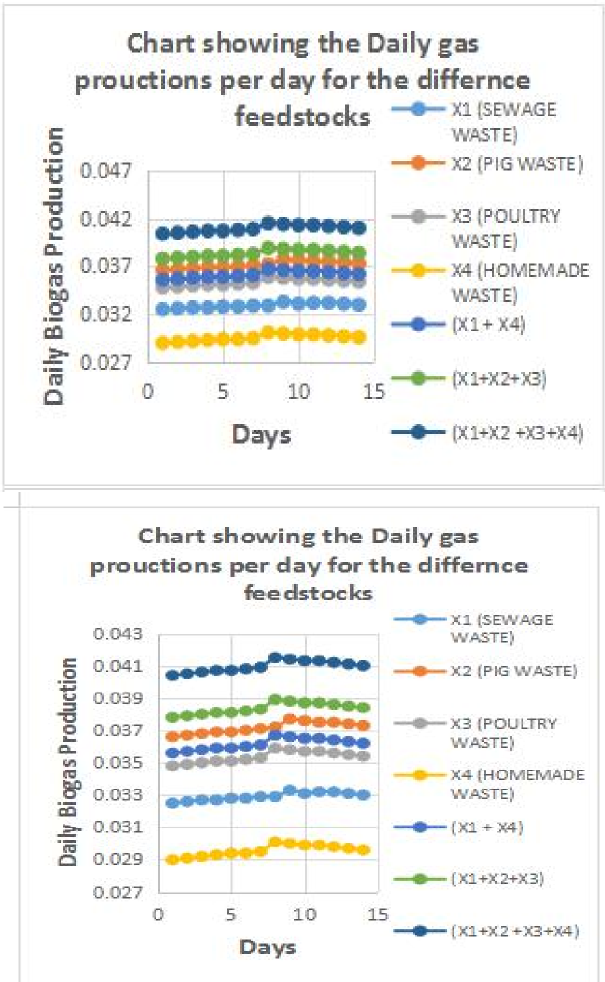

Fig 1: Showing the Relationship between the gas production rates of the different feedstocks

Fig 1 above summarizing the daily biogas production rates for all feedstocks used is presented below. The data of the daily biogas production rates for all the feedstocks were recorded over a 3-Month Period, also the focus was only on the 14days of maximum production period. The figure below also shows the methane and Carbon dioxide percentages of the gases produced by each sample (feedstock).

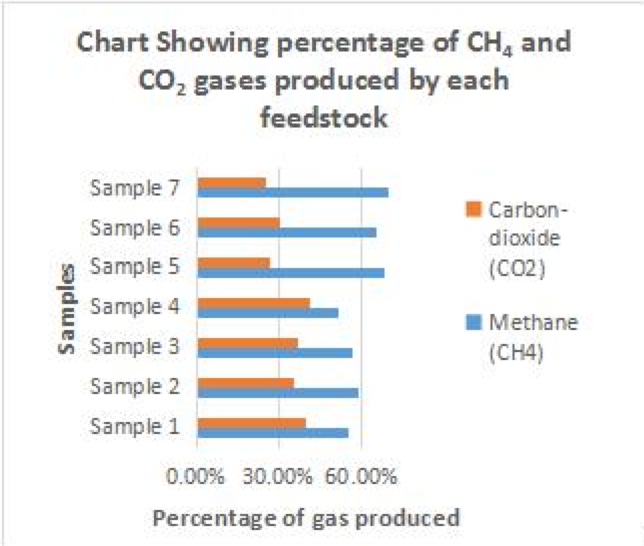

Fig 2: Figure showing the comparison between metane gas and CO_2 gas in each of the sample

Model Derivation:
Let the biogas produce by each of the feedstocks / substrates be represented by X1, X2, X3, and X4; Where,
X1 = S and represents the sewage feedstocks.
X2 = A1 and represents the pig waste.
X3 = A3 and represents the poultry wastes.
X4 =F1 and represents the combined homemade food waste (pineapple and watermelon peels).
Also let the methane content of the biogas from each of the feedstocks be given as M1, M2, M3, and M4 for X1, X2, X3, and X4, respectively.

Following the above, the complete model equation will become:

Maximize G = M1X1+ M2X2+ M3X3 + M4X4 (1)

Subject to:

$20\,X1 \leq 40$days
$7\,X2 \leq 30$days
$7\,X3 \leq 25$days
$10\,X4 \leq 30$days
$20x1 + 7x2 + 7x3 \leq 40$days
$20x1 + 10x4 \leq 40$days
$20x1 + 7x2 + 7x3 + 10x4 \leq 40$days
$X1, X2, X3, X4 \geq 1$

Model Equation with Methane content of the gas produced: the percentages of methane gotten from each of the feedstock as seen in the table 3 is as follows; M1 = 54.8%, M2 = 58.7%, M3 =56.6%, and M4 = 51.7%.

Substituting these into the equation (1), the equation become

Maximize G = 54.8X1 + 58.7X2 + 56.6X3 + 51.7X4 (2)

Subject to:

$20X1 \leq 40$days
$7X2 \leq 30$days
$7X3 \leq 25$days
$10X4 \leq 30$days
$20X1 + 10X4 \leq 40$days
$20X1 + 7X2 + 7X3 \leq 40$days
$20X1 + 7X2 + 7X3 + 10X4 \leq 40$days
$X1, X2, X3, X4 \geq 1$

Samples Classifications: Below is the Classification and nomenclature given to each of the samples (feedstocks) used in the experiment:
SAMPLES:
Sample 1 (X1): Biogas from sewage
Sample 2 (X2): Biogas from pig waste
Sample 3 (X3): Biogas from poultry waste
Sample 4 (X4): Biogas from homemade organic food waste
Sample 5 (X1 + X4) - Biogas from homemade organic food waste and sewage
Sample 6 (X1 + X2 + X3) - Biogas from poultry waste, pig waste and sewage
Sample 7 (X1 + X2 + X3 + x4) – Biogas from Biogas from homemade organic food wastes

(pineapple and watermelon), Pig waste, Poultry waste and sewage.

Average Daily Biogas production: Below is the average Biogas produced within the 14 highest production periods for each of the feedstocks:
Sample 1- 0.0329 m3
Sample 2 - 0.0372 m3
Sample 3- 0.0354 m3
Sample 4- 0.0296 m3
Sample 5- 0.0362 m3
Sample 6- 0.0384 m3
Sample 7- 0.0410 m3

The average Methane gas produced by each feedstock as well as other Gases is as shown in the table below:

3.3.4 Model Equation with only the average volume of gas is shown below:

Maximize G = 0.0329(x1)+0.0372(x2)+0.0354(x3)+0.0296(x4) (3)

Subject to:
$20X1 \leq 40$days
$7X2 \leq 30$days
$7X3 \leq 25$days
$10X4 \leq 30$days
$20X1 + 10X4 \leq 40$days
$20X1 + 7X2 + 7X3 \leq 40$days
$20X1 + 7X2 + 7X3 + 10X4 \leq 40$days
$X1, X2, X3, X4 \geq 1$

The Final Model equation (a product of the volume of gas produced and the methane content of the gas) is derived when we multiply the average daily volume of gas produced with the actual methane content of each feedstock as shown below:

Maximize G = 54.5 * 0.0329(x1) + 58.7*0.0372(x2) + 56.6 * 0.0354(x3) + 51.7 * 0.0296(x4) (4)

Subject to:
$20X1 \leq 40$days
$7X2 \leq 30$days
$7X3 \leq 25$days
$10X4 \leq 30$days
$20X1 + 10X4 \leq 40$days
$20X1 + 7X2 + 7X3 \leq 40$days
$20X1 + 7X2 + 7X3 + 10X4 \leq 40$days
$X1, X2, X3, X4 \geq 1$

Therefore, by simplifying the above equation, we will have:

Maximize G = 1.803x1+2.184x2+2.004x3+1.530x4 (5)

Subject to:
$20X1 \leq 40$days
$7X2 \leq 30$days
$7X3 \leq 25$days
$10X4 \leq 30$days
$20X1 + 10X4 \leq 40$days
$20X1 + 7X2 + 7X3 \leq 40$days
$20X1 + 7X2 + 7X3 + 10X4 \leq 40$days
$X1, X2, X3, X4 \geq 1$

Solving the model Using Simplex Method

Table 3: Showing Model Presentation before solving.

Variables	
X1	0
X2	0
X3	0
X4	0
Objective	
Maximize	0
Constraints	
	1
	2
	3
	4
	5
	6
	7

From the above, it was seen that variables were placed at zero (0) initially and also the objective function was zero (0). It also defines the initial characteristics of Constraints of the model. In other words, we can simply say that it describes the initial conditions of the model before analysis.

Table 4: Showing Model Results after solving.

Variables	
X1	0
X2	4.285714
X3	1.428571
X4	1.78E-16
Objective	
Maximize	12.22286
Contraints	
	1
	2
	3
	4
	5
	6
	7

After analyzing the model using simplex method in the excel solver, we obtain a maximum value of 12.22286 which represents the value of our Objective function: which is the maximum value of methane gas obtainable as long as our combination of feedstocks are concerned under the constraint conditions as well as the assumptions for the model. Apart from the maximum values, it also shows the combination of the variables that will give this maximum value which are as follows: 4.285714, 1.428571, and 1.78E-16 for X2, X3, and X4 respectively. This means that Pig dung waste will be the most significant variable when trying to get the maximum methane gas production from the combination of wastes used in the model, followed by the poultry dung waste and a very negligible portion of the home food waste, while the sewage waste has no significant at all when trying to maximize the gas production. Finally, the analysis showed how the production of the maximum value of methane gas can impact on the constraint conditions.

IV. CONCLUSION

This study has demonstrated the potential of using various waste sources—sewage, pig, poultry, and homemade (watermelon and pineapple) wastes—as substrates for biogas production through anaerobic digestion. The controlled laboratory experiments with mini-biodigesters revealed that the combination of multiple feedstocks generally resulted in higher biogas yields and methane content compared to individual substrates. Specifically, the combination of sewage, pig, poultry, and homemade waste achieved the highest biogas production rate and methane content, underscoring the benefits of mixed feedstocks. The mathematical modeling and optimization using the Simplex Method provided clear guidance on the most efficient substrate combinations, with the optimal feedstock mix involving primarily pig and poultry waste. These findings suggest that integrating different waste sources can significantly enhance biogas production efficiency, offering a viable solution for sustainable energy generation and waste management. This research therefore highlights the importance of substrate selection and optimization in anaerobic digestion processes. It underscores the potential of biogas as a renewable energy source that can help reduce dependence on fossil fuels while managing organic waste effectively. Future studies could expand on these findings by exploring larger-scale applications and evaluating the economic feasibility of biogas production from mixed waste sources.

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VI. REFERENCES

[1]. W. J. Van Nes, and T. D. Nhete, (2007). Biogas for a better life. *Renewable Energy World*, July–August 2007.

[2]. T. Bond, and M. R. Templeton. (2011). History and future of domestic biogas plants in the developing world. Energy for Sustainable Development, 15, 347-354.

[3]. J. Bogner, R. Pipatti, S. Hashimoto, C. Diaz, K. Mareckova, L. Diaz, P. Kjeldsen, S. Monni, A. Faaij, and Q. Gao, (2008). Mitigation of global greenhouse gas emissions from waste: conclusions and strategies from the Intergovernmental Panel on Climate Change (IPCC) Fourth Assessment Report. Working Group III (Mitigation). *Waste Management and Research*, 26, 11-32.

[4]. FAO (1996). System Approach to Biogas Technology.

[5]. S. Yadvika, T. R. Sreekrishnan, S. Kohli and V. Rana, (2004). Enhancement of biogas production from solid substrates using different techniques—a review. *Bioresource Technology*, 95, 1-10.

[6]. A. O. Atilade, O. K. Onanuga and J. O. Coker (2015). Comparative Study of Biogas Generation From Chicken Waste, Cow Dung, And Pig Waste Using Constructed Plastic Bio Digesters. ICASTOR Journal of Engineering Vol. 8, No. 1 (2015) 31 – 37.

[7]. S. T, Ubwa, K. Asemave, B. Oshido, and A. Idoko. Preparation of Biogas from Plants and Animal Waste. International Journal of Science and Technology Volume 2 No. 6, June, 2013.

[8]. M. E. Oseji, G. R. Ana, and A. A. Sokan-Adeaga. (2017). Evaluation of Biogas Yield and Microbial Species from Selected Multi-biomass Feedstocks in

Nigeria. London Journal of Research in Science: Natural and Formal. Volume 17 | Issue 1 | Compilation 1.0.

[9]. A. O. Ojikutu, and O. O. Osokoya. (2014). Evaluation of Biogas Production from Food Waste. The International Journal of Engineering and Science (IJES). ||Volume|| 3 ||Issue|| 01 || Pages || 01-07 || 2014 || ISSN (e): 2319 – 1813 ISSN (p): 2319 – 1805.

[10]. F. Di Maria, C. Micale, A. Sordi, and G. Cirulli. (2012). Co-digestion of sewage sludge and organic fraction of municipal solid waste. Conference: SIDISA at Milano.

[11]. K. H. Chua, W. L. Cheah, C. F. Tan, and Y. P. Leong, (2013). Harvesting biogas from wastewater sludge and food waste. 4th International Conference on Energy and Environment 2013 (ICEE 2013). IOP Conf. Series: Earth and Environmental Science 16 (2013) 012118. Doi:10.1088/1755-1315/16/1/012118.

[12]. H. I. Asinyetogha. (2016). Analyses of Anaerobic Batch Digestion of Municipal Solid Waste in the Production of Biogas Using Mathematical Models. Energy and Environment Research; Vol. 6, No. 1; 2016 ISSN 1927-0569 E-ISSN 1927-0577. Published by Canadian Center of Science and Education.

[13]. D. Batstone, J. Keller, I. Angelidaki, S. Kalyuzhnyi, S. Pavlostathis, A. Rozzi, Sanders W., Siegrist H. and Vavilin V. (2002). Anaerobic digestion model No 1 (ADM1). Water Science and Technology: a journal of the International Association on Water Pollution Research. 45. 65-73.

[14]. O. R. Onosakponome, J. O. Ademiluyi and C. Odenigbo. Modeling of biogas production by anaerobic digestion for the generation of electricity. (awaiting publication)

[15]. M. E. Lopes, R. C. Oliveira, and M. A. S. Rodrigues (2015) Optimization of biogas production from multiple feedstocks through co-digestion. Renewable Energy Journal, Volume 75, ISSN: 0960-1481.

[16]. H. N. Chanakya, G. M. Maheswarappa, and R. N. K. Goud. (2018) Biogas production from co-digestion of multiple organic wastes: A review. Bioresource Technology, Volume 265, ISSN: 0960-8524

[17]. S. Patil, M. Stöckl, and H. J. Vogel. Synergistic effect of co-digestion on biogas production from mixed organic substrates (2016) - Waste Management, Volume 47, ISSN: 0956-053X.

[18]. K. S. Rao, A. K. Sarada, and P. R. Babu. Biogas production from co-digestion of multiple organic wastes: An integrated approach (2017) - Environmental Science and Pollution Research, Volume 24, ISSN: 0944-1344.

[19]. P. S. Kumar, S. S. Smitha and S. K. Khanal. Enhanced biogas production through co-digestion of multiple feedstocks: A review (2021) - Renewable and Sustainable Energy Reviews, Volume 144, ISSN: 1364-0321.

[20]. R. S. Yuan, Y. H. Chang, and C. Y. Chang. Biogas production from co-digestion of sewage sludge with various organic wastes (2019) - Journal: Journal of Environmental Management, Volume 233, ISSN: 0301-4797. https://www.healthline.com/health/healthy-home-guide/sewer-gas

[21]. A. T. Prakash, S. K. Sharma, and R. R. Soni. Co-digestion of multiple organic wastes for enhanced biogas production: A techno-economic analysis (2020) - Energy Conversion and Management, Volume 209, ISSN: 0196-8904.

[22]. J. P. Singh, A. K. Jha, and S. K. Sharma. Biogas production from co-digestion of multiple feedstocks: A comparative study (2018) - Journal: International Journal of Energy Research, Volume 42, ISSN: 0363-907X.

[23]. S. R. Prasad, R. K. Patel, and S. K. Tyagi. Integration of multiple feedstocks for biogas production: A feasibility study (2017) - Journal: Waste and Biomass Valorization, Volume 8, ISSN: 1877-2641.

[24]. S. K. Shukla, P. N. Singh, and S. K. Tyagi. Biogas production from co-digestion of multiple substrates: A review on process challenges and strategies (2019) - Journal: Journal of Environmental Chemical Engineering, Volume 7, ISSN: 2213-3437

[25]. Sebola, Rebecca & Tesfagiorgis, Habtom & MUZENDA, EDISON. (2015). Methane Production from Anaerobic Co-digestion of Cow Dung, Chicken Manure, Pig Manure and Sewage Waste. 10.13140/RG.2.1.4997.5760.