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Simplified Octahedral Shear Stress Theory for Plane Elements

[1]Adah, Edward I., [2]Anyin, Peter. B., [3]Eludire, Oluwadamilare O., [4]Ukya, Terhinde J.,

[1,2,4]Department of Civil and Environmental Engineering, University of Calabar, Nigeria

[3]Department of Agricultural and Bio-Resources Engineering, University of Calabar, Nigeria

Corresponding Author: edwardadah@unical.edu.ng

Abstract: *The Tresca and von Misses yield criteria are generally suitable and widely used criteria for ductile materials with even the von Misses gaining more applicability. However, it is still limited in plate analysis especially for plates with one free edge. Therefore, this work aims at developing a new and simplified general permissible stress equation based on octahedral shear stress theory for plane element especially rectangular plates. This is done by modifying and improving the octahedral yield theory to obtain the new general mathematical model for yield criterion of thin isotropic rectangular plates as a function of the stress factor and vector parameter n_1. Based on this general formulated model, specific mathematical models based on edge condition are obtained using the polynomial displacement shape profile for various plate types. From the result obtained, the permissible stress for the considered plate types are 244MPa. These values are less than the yield stress of mild steel of 250MPA. Also, a comparison with existing works in literature gave a percentage difference of 2.45% with the present work been upper bound and closer to the yield value. This implies that the formulated models are adequate. These new mathematical models will enhance quick prediction of the permissible stress by analysts for safe design of plated structures.*

Keywords: *Plane element, rectangular plates, octahedral shear stress, stress factor, permissible stress*

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I. INTRODUCTION

The relevance of stress analysis in structures is paramount to structural safety and economic management. Ross (1987) states that the combination of stresses and strain that initiate yield in a material need to be considered because the initiation of yield is most times related to the ultimate failure of the structure or material. The author further mentioned that the theories of yield are related to the triaxial principal stress system, such that, $\sigma_1 > \sigma_2 > \sigma_3$. Where σ_1 and σ_3 are the maximum and minimum principal stress respectively and σ_2 is the minimax principal stress. According Chatti (2019), yield criteria are specific to different types of materials and are used to predict when yielding will occur under various loading conditions. Yield criteria are mathematical expressions that describe the relationship between the applied stress in a material and the beginning of plastic deformation. Lee (1977) assumed that the plastic behavior of the material is independent of time and temperature. Yielding occurs when a material's stress exceeds a certain threshold, known as the yield strength. Muhammad Rehan (2023) investigated yield in materials and observed that yield criteria help engineers predict the point at which materials will start to deform plastically.

Some of the elastic failure criteria are; the maximum principal stress theory, maximum principal strain theory, total strain energy theory, Tresca Criterion, and Octahedral shear stress theory or Von Mises criterion. Benham & Warnock, (1976) states that a yield criterion such as that of Tresca or von Mises which is based on principal stress difference would seem to be the most logical for use in metallic materials. This view is collaborated by Moy (1981), Ross (1987), Lee (1977), and Save et al. (1997). This view is more acceptable because of the fact that shear controls yield, and which can lead to the accurate prediction of the beginning of yield in ductile materials. Yu et al. (2009) proposed a new yield criterion which they claimed is newly developed and performs better than the existing ones and is ready for application. He summarized the development of mechanical models for various yield criteria and the applications of the unified strength theory to engineering. Oliveiraet al. (2021) stated that based on von Mises yield criterion, the ratio between the yield stresses in simple shear and in uniaxial tension is the same irrespective of the material. They presented a numerical study which reveals that even for one of the simplest deep drawing processes, namely the forming of a cylindrical cup, the yielding description influences the predictions of the plastic strains and the final profile of the part.

Even though the von Misses or octahedral criterion is more better and in popular use especially for ductile materials, it is still limited in plane structure analysis such as plates especially for plates with one free edge. Also, the use of polynomial shape profiles in this analysis is not available in literature to the authors' knowledge. Therefore, this work aims at modifying and improving this octahedral yield theory by obtaining new general mathematical model for yield criterion of thin rectangular plates as a function of the stress factor and vector parameter n_l. Based on this general formulated model, specific mathematical models based on edge conditions are obtained using the polynomial displacement shape profile for various plate types. This new mathematical model will enhance quick prediction of the permissible stress by analysts for safe design of plated structures.

According to Ibearugbulem et al. (2011), the sequence of edge arrangement of a plate is as shown in Fig. 1. Also, the naming of a plate based on edge conditions is as shown in Fig. 2.

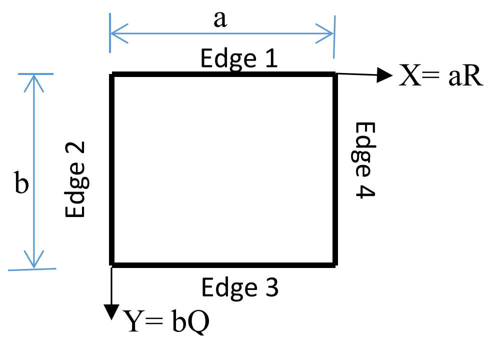

Fig. 1: Edge numbering of rectangular plate

Fig. 2 shows a plate simply supported (S) at edges 1 and 4, clamped (C) at edge 2, free (F) at edge 3 named SCFS plate type.

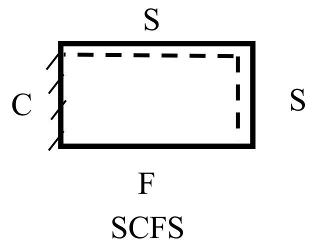

Fig. 2.0: A SCFS rectangular plate

II. THEORETICAL FORMULATION OF THE EQUATION

The octahedral shearing stress theory by Mises - Hacky or Von Mises in short, predicts that failure will occur by yielding when the octahedral shear stress at a point reaches the octahedral in a specimen made from the same material as the structure, when the specimen is subjected to simple uniaxial test (Ross, 1987; Benhan and Warnock, 1979).

According to them, for a general state of stress, τ_{oct} is given as

$$\tau_{oct} = \frac{1}{3}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau^2_{xy} + \tau^2_{xz} + \tau^2_{yz}\right)\right]^{½} \quad (1)$$

But from simple tension test,

$$\tau_{oct} = 0.47\, f_y \quad (2)$$

Where f_y is yield stress

Equating Eqn (2) to Eqn (1) and considering the safety of the structure

$$\frac{1}{3}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau^2_{xy} + \tau^2_{xz} + \tau^2_{yz}\right)\right]^{½} = 0.47 f_y \quad (3)$$

$$\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau^2_{xy} + \tau^2_{xz} + \tau^2_{yz}\right)\right]^{½} = 1.41 f_y \quad (4)$$

$$\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau^2_{xy} + \tau^2_{xz} + \tau^2_{yz}\right)\right] = 2 f_y^2 \quad (5)$$

For a 2 – dimensional problem

$\sigma_z = \tau_{xz} = \tau_{yz} = 0$

$$(\sigma_x - \sigma_y)^2 + \sigma_y^2 + \sigma_x^2 + 6\tau_{xy}^2 = 2{f_y}^2 \quad (7)$$

$$\sigma_x^2 - 2\sigma_x\sigma_y + \sigma_y^2 + \sigma_y^2 + \sigma_x^2 + 6\tau_{xy}^2 = 2{f_y}^2 \quad (8)$$

$$2(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2) = 2{f_y}^2 \quad (9)$$

$$\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2 = {f_y}^2 \quad (10)$$

$$\sigma_x^2\left(1 - \frac{\sigma_y}{\sigma_x} + \frac{\sigma_y^2}{\sigma_x^2} + 3\frac{\tau_{xy}^2}{\sigma_x^2}\right) = {f_y}^2 \quad (11)$$

$$\sigma_x^2\,(1 - m_1 + m_1^2 + 3m_2^2) = {f_y}^2 \quad (12)$$

where

$$m_1 = \frac{\sigma_y}{\sigma_x},\; m_2 = \frac{\tau_{xy}}{\sigma_x} \quad (13)$$

The stress in a 2-dimensional plane element expressed in non-dimensional parameters are given by Adah et al. (2023) as

$$\sigma_x = \frac{-EASt}{(1-\upsilon^2)a^2}\left(\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{ꙅ^2}\frac{\partial^2 h}{\partial Q^2}\right) \quad (14)$$

$$\sigma_y = \frac{-EASt}{(1-\upsilon^2)a^2}\left(\upsilon\frac{\partial^2 h}{\partial R^2} + \frac{1}{ꙅ^2}\frac{\partial^2 h}{\partial Q^2}\right) \quad (15)$$

$$\tau_{xy} = G\gamma_{xy} = \frac{-EASt(1-\upsilon)}{2(1-\upsilon^2)ꙅ\, a^2}\frac{\partial^2 h}{\partial R\partial Q} \quad (16)$$

Where

E is Young Modulus of elasticity, υ is Poisson ration, h is the shape profile of the element, S is the thickness of the plate from the middle fibre to the extreme surface is is half the thickness of the entire plate, hence S = 0.5, t is the plate thickness, ꙅ is the aspect ratio expressed as the ratio of the length of the width of the plate b, to the length of the plate a, (b/a), A is the amplitude of displacement, R and Q are the non-dimensional axes corresponding to x- and y- axes respectively.

Substituting Eqns (14) to (16) into Eqn (13) yields

$$m_1 = \frac{\upsilon\frac{\partial^2 h}{\partial R^2} + \frac{1}{ꙅ^2}\frac{\partial^2 h}{\partial Q^2}}{\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{ꙅ^2}\frac{\partial^2 h}{\partial Q^2}} \quad (17)$$

$$m_2 = \frac{\frac{1-\upsilon}{2s}\;\frac{\partial^2 h}{\partial R\partial Q}}{\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{ꙅ^2}\frac{\partial^2 h}{\partial Q^2}} \quad (18)$$

Equations (18) and (19) can be re-written as

$$m_1 = \frac{n_2}{n_1} \quad (19)$$

$$m_2 = \frac{1-\upsilon}{2ꙅ}\frac{n_3}{n_1} \quad (20)$$

Where the plane factors n, are given as

$$n_1 = \frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{ꙅ^2}\frac{\partial^2 h}{\partial Q^2} \quad (21)$$

$$n_2 = \upsilon\frac{\partial^2 h}{\partial R^2} + \frac{1}{ꙅ^2}\frac{\partial^2 h}{\partial Q^2} \quad (22)$$

$$n_3 = \frac{\partial^2 h}{\partial R\partial Q} \quad (23)$$

Substituting Equation (21) to (23) into Equation (12) yields

$$\sigma_x^2\left(1 - \frac{n_2}{n_1} + \frac{n_2^2}{n_1^2} + 3\left[\frac{(1-\upsilon)}{2ꙅ}\right]^2\frac{n_3^2}{{n_1}^2}\right) = {f_y}^2 \quad (24)$$

$$\sigma_x^2 = \frac{{f_y}^2}{1 - \frac{n_2}{n_1} + \frac{n_2^2}{n_1^2} + \frac{(1-\upsilon)^2}{4ꙅ^2}\frac{n_3^2}{n1^2}} \quad (25)$$

$$\sigma_x = \sqrt{\left[\frac{{f_y}^2}{1 - \frac{n_2}{n_1} + \frac{n_2^2}{n_1^2} + \frac{3(1-\upsilon)^2}{4ꙅ^2}\frac{n_3^2}{{n_1}^2}}\right]} \quad (26)$$

$$\sigma_x = \frac{f_y}{\sqrt{1 - \frac{n_2}{n_1} + \frac{n_2^2}{n_1^2} + \frac{3(1-\upsilon)^2}{4ꙅ^2}\frac{n_3^2}{{n_1}^2}}} \quad (27)$$

$$\sigma_x = \frac{f_y}{F_o} \quad (28)$$

Where the Octehedral stress factor F_o is given as

$$F_o = \sqrt{1 - \frac{n_2}{n_1} + \frac{n_2^2}{n_1^2} + \frac{3(1-\upsilon)^2}{4ꙅ^2}\frac{n_3^2}{{n_1}^2}} \quad (29)$$

A. Evaluation of Plane Factors 'n-Values' and Formulation of Stress Factor Equations

The various n-values (that is, n_1, n_2, and n_3) for the different plate types will be evaluated using the polynomial displacement shape profiles in Table 1.

Table 1: The polynomial displacement shape profiles

Plate Type	Shape Profile, h
SSFS	$(R-2R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
SCFS	$(1.5R^2-2.5R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
CSFS	$(R-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$
CCFS	$(1.5R^2-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$
SCFC	$(R^2-2R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
CCFC	$(R^2-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$

(Ibearugulem, et al. 2014); S-Simply supported edge, C - Clamped edge, F - Free edge

Where

$R = X/a$, $0 \le R \le 1$; $Q = Y/b$, $0 \le R \le 1$

a - plate dimension (length) along X-axis, b - is plate dimension (Width) along Y-axis

The n-values for the various plate types will be evaluated as follows.

From Table 1, an SSFS plate have a shape profile as

$$h = (R - 2R^3 + R^4)(\frac{7}{3}Q - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5) = h_x * h_y \quad (30)$$

$$\frac{\partial^2 h}{\partial R^2} = \frac{\partial^2 h_x}{\partial R^2} * h_y = (-12R + 12R^2)(\frac{7}{3}Q - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5) \quad (31a)$$

$$\frac{\partial^2 h}{\partial Q^2} = h_x * \frac{\partial^2 h_y}{\partial Q^2} = (R - 2R^3 + R^4)(-20Q + 40Q^2 - 20Q^3) \quad (32b)$$

$$\frac{\partial^2 h}{\partial R \partial Q} = \frac{\partial h_x}{\partial R} * \frac{\partial h_y}{\partial Q} = (-6R^2 + 4R^3)(\frac{7}{3} - 10Q^2 + \frac{40}{3}Q^3 - 5Q^4) \quad (33c)$$

Substitute Equations (32) into Equations (21)- (23) yields

$$n_1 = \left[(-12R + 12R^2)(\frac{7}{3}Q - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5) + \frac{\upsilon}{\text{ƨ}^2}(R - 2R^3 + R^4)(-20Q + 40Q^2 - 20Q^3)\right] \quad (34)$$

$$n_2 = \left[\upsilon(-12R + 12R^2)(\frac{7}{3}Q - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5) + \frac{1}{\text{ƨ}^2}(R - 2R^3 + R^4)(-20Q + 40Q^2 - 20Q^3)\right] \quad (35)$$

$$n_3 = (1 - 6R^2 + 4R^3)(\frac{7}{3} - 10Q^2 + \frac{40}{3}Q^3 - 5Q^4) \quad (36)$$

At the point of maximum deflection, R = 0.5, Q = 1. Substitute these values of R and Q in Equation (34) to Equation 36), we have

$$n_1 = -4 \quad (37)$$

$$n_2 = -4\upsilon \quad (38)$$

$$n_3 = 0 \quad (39)$$

Substituting Equation (37) to Equation (39) into Equation (29) yields the stress factor as Equation (40)

$$F = \left[1 - \upsilon + \upsilon^2\right]^{\frac{1}{2}} \quad (40)$$

For these plates with a free edge, the maximum deflection will occur at R = 0.5, Q = 1. Therefore, the rest of the of the five plate types contain in Table 1, were evaluated.

The stress factor equations for the six plates are presented in Table 3.

B. Numerical Example

Consider a structural steel square plate with the following properties. $\upsilon = 0.3$ and $f_y = 250$MPa.

The numerical results obtain from yield criterion equations in Table 3 are presented in Table 4.

III. RESULTS AND DISCUSSIONS

The general formulated octahedral yield criteria equation, that is the applied stress equation from this analysis is presented in Table 2. This equation is the ratio of the yield stress over the stress factor. The developed stress factor equations for the different plate types based of the edge conditions of the plates are presented in Table 3. The stress factor equation is a function of the Poisson ratio, which is a property of the material. This means the type of materials determines the magnitude of stress it can bear.

Table 2: General Formulated Octahedral Yield Criterion Equations

SN	DESCRIPTION	EQUATIONS
1	Applied Stress	$\sigma_x = \frac{f_y}{F_o}$
2	Stress Factor	$F_o = \sqrt{\left[1 - \frac{n_2}{n_1} + \frac{n_2^2}{n_1^2} + \frac{3(1-\upsilon)^2}{4ƨ^2}\frac{n_3^2}{{n_1}^2}\right]}$
3	n-values	$n_1 = \left(\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right)$
		$n_2 = \left(\upsilon\frac{\partial^2 h}{\partial R^2} + \frac{1}{ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right)$
		$n_3 = \left(\frac{\partial^2 h}{\partial R \partial Q}\right)$

Table 3: Octahedral Stress Factor, F_o, Equations

Plate Type	$\sigma_x = \frac{f_y}{F_o}$
	$F_o = \sqrt{\left[1 - \frac{n_2}{n_1} + \frac{n_2^2}{n_1^2} + \frac{3(1-\upsilon)^2}{4ƨ^2}\frac{n_3^2}{{n_1}^2}\right]}$
SSFS	$[1 - \upsilon + \upsilon^2]^{\frac{1}{2}}$
SCFS	$\left[1 - \upsilon + \upsilon^2 + \frac{0.001302083323(1 - 2\upsilon + \upsilon^2)}{ƨ^2}\right]^{\frac{1}{2}}$
CSFS	$[1 - \upsilon + \upsilon^2]^{\frac{1}{2}}$
CCFS	$\left[1 - \upsilon + \upsilon^2 + \frac{0.001302083333(1 - 2\upsilon + \upsilon^2)}{ƨ^2}\right]^{\frac{1}{2}}$
SCFC	$[1 - \upsilon + \upsilon^2]^{\frac{1}{2}}$
CCFC	$[1 - \upsilon + \upsilon^2]^{\frac{1}{2}}$

The numerical values of stress factor, F_o, and the applied stress, σ_x, for aspect ratio of one for the twelve plate types under consideration based on the initial yield stress of 250MPa for mild steel are presented in columns 2 and 3 respectively of Table 4. Based on the results shown, it is seen that the octahedral stress factor for plate types with one free edge are less that one resulting in applied stress values for these plate types to be greater that the material yield stress of 250MPa. While F_o values for plate types with no free edge are equal to or greater than one resulting in applied stress values less than the material yield stress with the exception of CCSS. The implication is that, the plate types with one free edge are liable to failure if nothing is done to remedy the stress. Therefore, in structural design, it is advisable to introduce a factor of safety to reduce the stress to allowable safe limits resulting in permissible stress, given as

$$\sigma_p = \frac{\sigma_x}{F_s} \quad (41)$$

Now applying a factor of safety of 1.15 is which the range of most factor of safeties used in limit state design, the permissible stress for the plate types with one free edge will reduce to 244MPa. This value is less than the material yield stress, therefore ensuring safety of the structures. With the introduction of these factors of safety the new formulated octahedral permissible stress equation (eqn 41) is adequate for analysis and design of plate structures to ensure safety, functionality and sustainability. To further validate this work, the allowable stress obtained from this work were compared with those values obtained by Adah et al. (2023) who used maximum shear stress theory approach in Table 5 for plates with one free edge. The maximum percentage difference was 2.46%. This value is small and within the acceptable limit in statistics. It also shows that the present values are upper bound to those compared with, indicating that present values are more closer to the yield stress of 250MPa than the compared values, confirming the adequacy of this present work.

Table 4: Octahedral Permissible Stress (σ_a)

Plate Type	$\sigma_p = \frac{\sigma_x}{F_s}$; $\upsilon = 0.3$, $\beta = \frac{b}{a} = 1$; $f_y = 250MPa$		
	F_o	σ_x(MPa)	σ_p(MPa)
SSFS	0.888819	281	244
SCFS	0.889178	281	244
CSFS	0.888819	281	244
CCFS	0.889178	281	244
SCFC	0.888819	281	244
CCFC	0.888819	281	244

Table 5: Comparison of allowable stress of this work with previous work.

Plate type	Present work	Adah et al. (2023)	%Difference
SSFS	244	238	2.46
SCFS	244	238	2.46
CSFS	244	238	2.46
CCFS	244	238	2.46
SCFC	244	238	2.46
CCFC	244	238	2.46

IV. CONCLUSSION

The present study has derived an improved octahedral permissible equation based on the octahedral shear stress theory. The new permissible equation has introduced a factor of safety to ensure that the applied stresses are below the material stress to avoid failure of the plated structure. Besides, the equation is very simple and provides an easy approach for analyst and designers to predict the permissible stress of plane elements such as plates. Also, the used of many plate types has proven that the equation is applicable to many plate types not covered here and to other plate materials other than mild steel.

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