Nnamdi Azikiwe University Journal of Civil Engineering (NAUJCVE)

Volume-3,Issue-1,pp-30-37

www.naujcve.com

Research Paper Open Access

Formulation of limit state deflection equation for thin rectangular steel plates analysis

[1*]Adah, Edward I., [2]Ibearugbulem, Owus M., [3]Arum, Chinwuba, [4]Anyin, Peter B.

[1,4]Department of Civil and Environmental Engineering, University of Calabar, Nigeria
[2]Department of Civil Engineering, Federal University of Technology, Owerri, Nigeria
[3]Department of Civil and Mining Engineering, University of Namibia, Namibia

Corresponding Author: *edwardadah@unical.edu.ng

Abstract: *The limit state is the ultimate point of a structural capacity beyond which it is assumed to have failed. The aim of this work is to develop a general limit state deflection equation for thin rectangular plates analysis, and then based on the polynomial shape profiles formulate specific equations for twelve plate types. This will be done by using the limit state amplitude equation to determine the limit state deflection equation in terms of stress factor, F, and the vector normal to principal x- and y- axes,. Numerical results obtained from the equations at point of maximum deflection are analyzed. A look at the limit state values shows that the plates with one free edge deflect more compared with those with no free edge. Also, the plate simply supported on three edges and free at one (SSFS) and plate clamped at one edge, simply supported at two edges and free at one edge (CSFS) deflected the highest (12.421mm). While the plate clamped on all four edges (CCCC) deflected the least (4.622mm). These observations agree with the practical behaviour of thin plates. Hence, an indication of the adequacy of these new equations for predicting limit state deflection values of rectangular plates. Also, it will help plate analysts to easily predict the limit of deflection and can easily design to avert fail of plated structures and avoid economic losses for sustainable advancement.*

Key words: *Limit State, Deflection, Stress factor, Rectangular plates, Specific equations*

Date of Submission: 24-01-2025 Date of acceptance: 02-02-2025

The magnitude of deflection varies from one structural boundary condition to the other. The nature of the boundary condition affects the magnitude of the deflection. The deflected shape profile has been expressed in terms of trigonometric, polynomial, or hyperbolic functions (Levy, 1942; Timoshenko, 1959; Ugural, 1999; Ventsel and Krauthermer, 2001; Chakarverty, 2001; Szilard, 2004, Ibearugbulem et al. 2014). Due to the difficulties of evaluating the trigonometric and hyperbolic functions, Ibearugbulem (2012) in his PhD research formulated the deflected shape profile using polynomial series, which gave rise to the polynomial shape profiles. These polynomial profiles are easy to evaluate and are documented in for twelve boundary conditions in Ibearugbulem et al. (2014) and will be used in this work.

The deflection is an important factor that needs to be controlled if structural failures due to deflection are to be averted. Hence, design codes specified allowable limits to guide deflection conditions. The aim of this work is to develop a general limit state deflection equation for thin rectangular plates analysis, and then based on the polynomial shape profiles formulate specific equations for twelve plate types. These equations will provide a guide on the safe limit deflection based on the predicted values from the new deflection expressions. The work will be beneficial to structural analysts and designers of plate structures as well as allied professional interested in the field of solid mechanics. It will also generate reliable data for the advancement of research for development.

II. FORMULATION OF GENERAL LIMIT STATE DEFLECTION EQUATION UNDER LARGE DEFLECTION `

The general deflection shape of a continuum is given by Ibearugbulem (2012)

$$w = Ah \qquad (1)$$

where A is the amplitude of deflection and h is the shape profile of the plate based on the plate type.
At limit state condition, $A = A_{ls}, h = h_{max}$. Hence, Equation (1) becomes

$$w = A_{ls} h_{max} \qquad (2)$$

Also, Adah et al. (2023) derived the limit state amplitude of deflection equation or the coefficient of deflection under large deflection as given in Equation (3)

$$A_{ls} = \frac{-f_y(1-\upsilon^2)}{0.5FEn_1}\frac{a^2}{t} \qquad (3)$$

Where

f_y is yield stress of steel, υ is Poisson ratio, a is the dimension of a plate along the X-axis, t is the plate thickness, E is the young modulus of elasticity, and F is the stress factor, given in Equation (4).

$$F = \left[1 + \frac{{n_2}^2}{{n_1}^2} - 2\upsilon\frac{n_2}{n_1} + \frac{(1-\upsilon-\upsilon^2+\upsilon^3)}{2\beta^2}\frac{{n_3}^2}{{n_1}^2}\right]^{\frac{1}{2}} \qquad (4)$$

where

$$\beta = \frac{b}{a} \quad \text{is aspect ratio} \qquad (5)$$

and n_1, n_2, and n_3 are given as

$$n_1 = \frac{\partial^2 h_x}{\partial R^2} * h_y + \frac{\upsilon}{\beta^2} * h_x * \frac{\partial^2 h_y}{\partial Q^2} \qquad (6)$$

$$n_2 = \upsilon\left(\frac{\partial^2 h_x}{\partial R^2} * h_y\right) + \frac{1}{\beta^2}\left(h_x * \frac{\partial^2 h_y}{\partial Q^2}\right) \qquad (7)$$

$$n_3 = \frac{\partial h_x}{\partial R} * \frac{\partial h_y}{\partial Q} \qquad (8)$$

Substituting Equation (3) into Equations (2) yields

$$w_{ls} = \frac{-f_y(1-\upsilon^2)h_{max}}{0.5FEn_1} \cdot \frac{a^2}{t} \qquad (9a)$$

Which can be written in compact form as

$$w_{ls} = \alpha_w \frac{a^2}{t} \qquad (9b)$$

Where

$$\alpha_w = \frac{-f_y(1-\upsilon^2)h_{max}}{0.5FEn_1} \qquad (10)$$

Equation (9) is the general limit state deflection equation for thin rectangular plates. The general limit state deflection equation is presented in Table 2.

A. Displacement shape Profiles for thin rectangular plates.

The polynomial displacement shape profiles of the two twelve plates considered for this study are presented in Table 1.

Table 1: The polynomial displacement shape profiles

Plate Type	Shape Profile, h
CCCC	$(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$
SSSS	$(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$
CSSS	$(R - 2R^3 + R^4)(1.5Q^2 - 2.5Q^3 + Q^4)$
CSCS	$(R - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$
CCSS	$(1.5R^2 - 2.5R^3 + R^4)(1.5Q^2 - 2.5Q^3 + Q^4)$
CCCS	$(1.5R^2 - 2.5R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$
SSFS	$(R-2R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$

SCFS	$(1.5R^2\text{-}2.5R^3\text{+}R^4)(\frac{7}{3}Q\text{-}\frac{10}{3}Q^3\text{+}\frac{10}{3}Q^4\text{-}Q^5)$
CSFS	$(R\text{-}2R^3\text{+}R^4)(2.8Q^2\text{-}5.2Q^3\text{+}3.8Q^4\text{-}Q^5)$
CCFS	$(1.5R^2\text{-}2R^3\text{+}R^4)(2.8Q^2\text{-}5.2Q^3\text{+}3.8Q^4\text{-}Q^5)$
SCFC	$(R^2\text{-}2R^3\text{+}R^4)(\frac{7}{3}Q\text{-}\frac{10}{3}Q^3\text{+}\frac{10}{3}Q^4\text{-}Q^5)$
CCFC	$(R^2\text{-}2R^3\text{+}R^4)(2.8Q^2\text{-}5.2Q^3\text{+}3.8Q^4\text{-}Q^5)$

Source: Ibearugbulem et al. (2014)

B. Formulation of Specific Limit State Deflection Equations for the Various Plate Types.

Formulation of Limit State Deflection Equation for CCCC Plate

The shape profile from Table 1 is

$$h = (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) = h_x * h_y \qquad (11)$$

$$\frac{\partial^2 h}{\partial R^2} = \frac{\partial^2 h_x}{\partial R^2} * h_y = (2 - 12R + 12R^2)(Q^2 - 2Q^3 + Q^4) \qquad (12a)$$

$$\frac{\partial^2 h}{\partial Q^2} = h_x * \frac{\partial^2 h_y}{\partial Q^2} = (Q^2 - 2R^3 + R^4)(2 - 12Q + 12Q^2) \qquad (12b)$$

$$\frac{\partial^2 h}{\partial R \partial Q} = \frac{\partial h_x}{\partial R} * \frac{\partial h_y}{\partial Q} = (2R - 6R^2 + 4R^3)(2Q - 6Q^2 + 4Q^3) \qquad (12c)$$

Substitute Equations (12) into Equations (6)- (8) yields

$$n_1 = \left[(2 - 12R + 12R^2)(Q^2 - 2Q^3 + Q^4) + \frac{\upsilon}{ƨ^2}(Q^2 - 2R^3 + R^4)(2 - 12Q + 12Q^2)\right] \qquad (13)$$

$$n_2 = \left[\upsilon(2 - 12R + 12R^2)(Q^2 - 2Q^3 + Q^4) + \frac{1}{ƨ^2}(Q^2 - 2R^3 + R^4)(2 - 12Q + 12Q^2)\right] \qquad (14)$$

$$n_3 = (2 - 6R^2 + 4R^3)(2 - 6Q^2 + 4Q^3) \qquad (15)$$

At point of maximum deflection, R = Q = 0.5. Substitute these values of R and Q in Equations (13) to (15), yields

$$n_1 = \left(-0.0625 - \frac{0.0625\upsilon}{ƨ^2}\right) \qquad (16)$$

$$n_2 = \left(-0.0625\upsilon - \frac{0.0625}{ƨ^2}\right) \qquad (17)$$

$$n_3 = 0 \qquad (18)$$

Substituting Equation (16) to (18) into Equation (4) yields the stress factor as Equation

$$F = \left[1 + \frac{\left(-0.0625ƨ^2\upsilon - 0.0625\right)^2}{\left(-0.0625ƨ^2 - 0.0625\upsilon\right)^2} - 2\upsilon\frac{\left(-0.0625ƨ^2\upsilon - 0.0625\right)}{\left(-0.0625ƨ^2 - 0.0625\upsilon\right)}\right]^{\frac{1}{2}} \qquad (19)$$

Based on similar approach, the other plate types were estimated.

Substituting Equations (16), and (19) into Equation (10) yields

$$\alpha_w \leq \frac{-f_y(1-\upsilon^2)h_{max}ƨ^2}{0.5E\left(-0.0625ƨ^2 - 0.0625\upsilon\right)} * \frac{1}{\sqrt{\left[1 + \frac{\left(-0.0625ƨ^2\upsilon - 0.0625\right)^2}{\left(-0.0625ƨ^2 - 0.0625\upsilon\right)^2} - 2\upsilon\frac{\left(-0.0625ƨ^2\upsilon - 0.0625\right)}{\left(-0.0625ƨ^2 - 0.0625\upsilon\right)}\right]}} \qquad (20)$$

Substituting Equation (20) into Equation (9b) yield

$$w_{ls} \le \frac{-f_y(1-\upsilon^2)h_{max}\text{ƨ}^2}{0.5E(-0.0625\text{ƨ}^2-0.0625\upsilon)} * \frac{1}{\sqrt{\left[1+\frac{(-0.0625\text{ƨ}^2\upsilon-0.0625)^2}{(-0.0625\text{ƨ}^2-0.0625\upsilon)^2}-2\upsilon\frac{(-0.0625\text{ƨ}^2\upsilon-0.0625)}{(-0.0625\text{ƨ}^2-0.0625\upsilon)}\right]}}\frac{a^2}{t} \quad (21)$$

Simplifying further, at the point of maximum deflection, R= Q = 0.5. Then

$$h_{max} = (0.5^2 - 2*0.5^3 + 0.5^4) * (0.5^2 - 2*0.5^3 + 0.5^4)$$

$$h_{max} = (0.0625)^2 = 0.00390625 \quad (22)$$

Substituting Equation (22) into Equations (20) and (21) yield

$$\alpha_w = \frac{-0.0078125f_y(1-\upsilon^2)\text{ƨ}^2}{E(-0.0625\text{ƨ}^2-0.0625\upsilon)} * \frac{1}{\sqrt{\left[1+\frac{(-0.0625\text{ƨ}^2\upsilon-0.0625)^2}{(-0.0625\text{ƨ}^2-0.0625\upsilon)^2}-2\upsilon\frac{(-0.0625\text{ƨ}^2\upsilon-0.0625)}{(-0.0625\text{ƨ}^2-0.0625\upsilon)}\right]}} \quad (23)$$

$$w_{ls} \le \frac{-0.0078125f_y(1-\upsilon^2)\text{ƨ}^2}{E(-0.0625\text{ƨ}^2-0.0625\upsilon)} * \frac{1}{\sqrt{\left[1+\frac{(-0.0625\text{ƨ}^2\upsilon-0.0625)^2}{(-0.0625\text{ƨ}^2-0.0625\upsilon)^2}-2\upsilon\frac{(-0.0625\text{ƨ}^2\upsilon-0.0625)}{(-0.0625\text{ƨ}^2-0.0625\upsilon)}\right]}}\frac{a^2}{t} \quad (24)$$

Equation (23) is the equation for the coefficient of limit state deflection for CCCC plate and Equation (24) is the limit state deflection equation for CCCC plate analysis.

In a similar way, the other plate types based on the shape profiles in Table 1 were estimated, and the results are presented Tables 3 for the twelve plate types considered. Note that for the plates with one free edge, maximum deflection will occur at R= 0.5 and Q=1.

C. Numerical Example

For the purpose of these analyses, the numerical parameters used are as follows. Considering a structural steel of grade A36: f_y = 250MPa, f_u = 400MPa, E = 200000MPa, a = 1m, υ = 0.3, t = 0.02m.

Now substituting these parameters in Equations (23) and (24) for CCCC plate yields the numerical results of limit state deflection coefficients and limit state deflection are presented in Table 4 and Table 5 respectively.

III. RESULT AND DISCUSSIONS

The results of the general limit state deflection equation are presented in Table 2. Therefore, for a plate not to fail, the deflection value should not exceed the one calculated from this equation. This equation provides a general guidance on the limiting deflection. Table 3 presents the limit state deflection equation for the specific rectangular plate types. These equations will predict the maximum deflection at which the limiting stress will occur for any given plate type. The advantage of these equations is that they can handle any plate aspect ratio and are applicable to several materials.

Table 2: General Limit State Deflection Equation

SN	DESCRIPTION	EQUATIONS
1	Limit State Deflection	$w_{ls} \le \frac{-f_y(1-\upsilon^2)h_{max}}{0.5EFn_1}\frac{a^2}{t}$

Table 3a: Coefficient of Limit State Deflection Equations, α_w, for Failure Analysis

Plate Type	$w_{ls} \leq \alpha_w \frac{a^2}{t}; \quad Ƨ = \frac{b}{a};$
	$\alpha_w = \frac{-f_y(1-\upsilon^2)h_{max}}{0.5EFn_1}$
CCCC	$\frac{-0.0078125f_y(1-\upsilon^2)Ƨ^2}{E(-0.0625Ƨ^2-0.0625\upsilon)} * \frac{1}{\sqrt{\left[1+\frac{(-0.0625Ƨ^2\upsilon-0.0625)^2}{(-0.0625Ƨ^2-0.0625\upsilon)^2}-2\upsilon\frac{(-0.0625Ƨ^2\upsilon-0.0625)}{(-0.0625Ƨ^2-0.0625\upsilon)}\right]}}$
SSSS	$\frac{-0.1953125f_y(1-\upsilon^2)Ƨ^2}{E(-0.9375Ƨ^2-0.9375\upsilon)} * \frac{1}{\sqrt{\left[1+\frac{(-0.9375Ƨ^2\upsilon-0.9375)^2}{(-0.9375Ƨ^2-0.9375\upsilon)^2}-2\upsilon\frac{(-0.9375Ƨ^2\upsilon-0.9375)}{-0.9375Ƨ^2-0.9375\upsilon}\right]}}$
CSSS	$\frac{-0.0781250f_y(1-\upsilon^2)Ƨ^2}{E(-0.375Ƨ^2-0.46875\upsilon)} * \frac{1}{\sqrt{\left[1+\frac{(-0.375Ƨ^2\upsilon-0.46875)^2}{(-0.375Ƨ^2-0.46875\upsilon)^2}-2\upsilon\frac{(-0.375Ƨ^2\upsilon-0.46875)}{(-0.375Ƨ^2-0.46875\upsilon)}\right]}}$
CSCS	$\frac{-0.0390625f_y(1-\upsilon^2)Ƨ^2}{E(-0.1875Ƨ^2-0.3125\upsilon)} * \frac{1}{\sqrt{\left[1+\frac{(-0.1875Ƨ^2\upsilon-0.3125)^2}{(-0.1875Ƨ^2-0.3125\upsilon)^2}-2\upsilon\frac{(-0.1875Ƨ^2\upsilon-0.3125)}{(-0.1875Ƨ^2-0.3125\upsilon)}\right]}}$
CCSS	$\frac{-0.03125f_y(1-\upsilon^2)Ƨ^2}{E(-0.1875Ƨ^2-0.1875\upsilon)} * \frac{1}{\sqrt{\left[1+\frac{(-0.1875Ƨ^2\upsilon-0.1875)^2}{(-0.1875Ƨ^2-0.1875\upsilon)^2}-2\upsilon\frac{(-0.1875Ƨ^2\upsilon-0.1875)}{(-0.1875Ƨ^2-0.1875\upsilon)}+\frac{0.00012207031252Ƨ^2(1-\upsilon-\upsilon^2+\upsilon}{(-0.1875Ƨ^2-0.1875\upsilon)^2}\right.}}$
CCCS	$\frac{-0.015625f_y(1-\upsilon^2)Ƨ^2}{E(-0.09375Ƨ^2-0.125\upsilon)} * \frac{1}{\sqrt{\left[1+\frac{(-0.09375Ƨ^2\upsilon-0.125)^2}{(-0.09375Ƨ^2-0.125\upsilon)^2}-2\upsilon\frac{(-0.09375Ƨ^2\upsilon-0.125)}{(-0.09375Ƨ^2-0.125\upsilon)}\right]}}$

Table 3b: Coefficient of Limit State Deflection Equations, α_w, for Failure Analysis

Plate Type	$w_{ls} \leq \alpha_w \frac{a^2}{t}; \quad Ƨ = \frac{b}{a};$
	$\alpha_w = \frac{-f_y(1-\upsilon^2)h_{max}}{0.5EFn_1}$
SSFS	$\frac{0.208333333f_y(1-\upsilon^2)}{E} \frac{1}{\sqrt{[1+\upsilon^2-2\upsilon^2]}}$

SCFS	$\frac{0.166666667f_y(1-\upsilon^2)}{E}\frac{1}{\sqrt{\left[1+\upsilon^2-2\upsilon^2+\frac{0.0008680555486(1-\upsilon-\upsilon^2+\upsilon^3)}{\mathcal{Z}^2}\right]}}$
CSFS	$\frac{0.208333333f_y(1-\upsilon^2)}{E}\frac{1}{\sqrt{[1+\upsilon^2-2\upsilon^2]}}$
CCFS	$\frac{0.166666667f_y(1-\upsilon^2)}{E}\frac{1}{\sqrt{\left[1+\upsilon^2-2\upsilon^2+\frac{0.0008680555556(1-\upsilon-\upsilon^2+\upsilon^3)}{\mathcal{Z}^2}\right]}}$
SCFC	$\frac{0.125f_y(1-\upsilon^2)}{E}\frac{1}{\sqrt{[1+\upsilon^2-2\upsilon^2]}}$
CCFC	$\frac{0.125f_y(1-\upsilon^2)}{E}\frac{1}{\sqrt{[1+\upsilon^2-2\upsilon^2]}}$

The general equations show that, the deflection is directly proportional to the square of the distance along the x-axis and inversely proportional to the thickness of the plate. It implies that the longer the span the higher the deflection and vice versa. Also, the higher the thickness of the plate the smaller or lower the deflection and vice versa. This is in conformity with the practical behaviour of structures, and a sign of the adequacy of the new equation. To understand the magnitude of the limit state deflection, numerical calculations for a 20mm steel plate was carried out as presented in Table 4. In the second and third columns of Table 4, the coefficient of the limit state deflection, α_w , and the numerical values of the limit state deflection respectively are presented for aspect ratio of one. This implies that beyond the limit state deflection values in column 3 of Table 4 for 20mm plate, the plate is liable to fail. A look at the limit state values shows that the plates with one free edge will deflect more compared with those with no free edge. Also, the SSFS and CSFS plates deflected the highest (12.421mm), while plate clamped on all four edges (CCCC) deflected the least (4.622mm). These observations are both logically and practically correct. Hence, an indication of the adequacy of these new equations for predicting limit state deflection values of rectangular plates. Besides, the equations are simple and easy to use by even those with just an average knowledge of plated structures. Since all that is needed is the dimensions of the plate (that is, length, breadth and thickness of the plate) with the support conditions at the edges. The equations are applicable to any rectangular thin plate material. In doing so, all that is needed is to substitute the Poisson ratio for that material into the equation for that plate type based on the edge conditions of the plate.

Table 4: Values of the Limit State Deflection Coefficient (α_w) and Limit State Deflection (w)

Plate Type	For, $Ƨ = \frac{b}{a} = 1$; $a = 1.0m, \upsilon = 0.3, t = 0.02m, f_y = 250MPa$		
	α_w	w_{ls} (m)	w_{ls} (mm)
CCCC	0.00009244	0.004622	4.622
SSSS	0.00015406	0.007703	7.703
CSSS	0.00013649	0.006825	6.825
CSCS	0.00011365	0.005683	5.683
CCSS	0.00012319	0.006160	6.160
CCCS	0.00010507	0.005253	5.253
SSFS	0.00024842	0.012421	12.421
SCFS	0.00019868	0.009934	9.934
CSFS	0.00024842	0.012421	12.421
CCFS	0.00019868	0.009934	9.934
SCFC	0.00014905	0.007453	7.453
CCFC	0.00014905	0.007453	7.453

IV. CONCLUSION

From the present study, the following conclusions are made as follows:

i. The general limit state deflection equation for rectangular plates under large deflection have been formulated.

ii. Specific equations based on twelve plate types have been formulated.

iii. The numerical values of the limit state deflection for the plate types are in compliance with logical reasoning and the practical behaviour of the plates.

iv. Therefore, the new equations are adequate and provide a quick means of predicting the limit state deflection of rectangular plates, and a good guide to analysts and designers to ensure safety of plated structures.

COMPETING INTERESTS

We declared that there are no competing interests.

REFERENCE

[1] Adah,E. I. (2023). "*Limit State Stability and Dynamic Analysis of Thin Rectangular Plates under Large Deflection*", PhD Dissertation submitted to Postgraduate School, Federal University of Technology Owerri.

[2] Chajes, A., (1974). "*Principles of Structural Stability Theory*". London: Prentice-Hall Inc.

[3] Chakraverty, S. (2009). "*Vibration of Plates*", (Taylor & Francis Group, London.

[4] Enem J. I., (2018). "*Geometrically Nonlinear Analysis of Isotropic Rectangular Thin Plates Using Ritz Method*". Ph.D. Thesis: Department of Civil Engineering, Federal University of Technology Owerri, Nigeria.

[5] Ibearugbulem, O.M., & Ezeh, J. C. (2012). "Instability of axially Compressed CCCC Thin Rectangular Plate Using Taylor's Series Shape Functions on Ritz Method", *Academic Research International*, 4(1), 346-351.

[6] Ibearugbulem, O.M., Ezeh, J.C., Ettu, L. O. (2014). "*Energy Methods in Theory of Rectangular Plates: Use of Polynomial Shape Functions*". Owerri, Liu House of Excellence Ventures.

[7] Ibearugbulem, O. M. (2012). "*Application of direct variational principles in elastic stability of rectangular flat thin plates*".Ph.D thesis submitted to Postgraduate School, Federal University of Technology, Owerri.

[8] Iyengar, N. G., (1988). "*Structural Stability of Columns and Plate*". Chichester, Ellis Horwood (1988).

[9] Levy, S., (1942). "Bending of rectangular plates with large deflections". *Technical notes: National Advisory Committee for Aeronautics (NACA)*, N0. 846 (1942).

[10] Moy, S.S. J. (1981). "Plastic Methods for Steel and Concrete Structures" Macmillian Press Ltd, Hong Kong.

[11] Oguaghamba, O. A., Ezeh, J. C., Ibearugbulem, O.M., Ettu, L. O., (2015). "Buckling and Postbuckling Loads Characteristics of All Edges Clamped Thin Rectangular Plate". The *International Journal of Engineering and Science*, 4(11), 55-61, (2015).

[12] Onodagu, P. D.(2018) "*Nonlinear Dynamic Analysis of Thin Rectangular Plates using Ritz Method*" Ph.D. Thesis, Federal Univ of Tech, Owerri, Nigeria.

[13] Onwuka, D. O., Ibearugbulem O. M. and Adah, E. I., (2016). "Stability analysis of axially compressed SSSS & CSCS plates using Matlab programming". *International Journal of Science and Technoledge*, 4(1), 66-71, (2016).

[14] Szilard, R. (2004). "*Theories and Application of Plate Analysis",* New Jersey: John Wiley & Sons.

[15] Timoshenko, S. P., Woinowsky-Krieger, S. (1959). "*Theory of plates and shells*", 2nd ed, McGraw-Hill, New York, (1959).

[16] Ugural, A.C. & Fenster, S.K. (2003). "*Advanced Strength & Applied Elasticity"*. 4th ed., New Jersey:Prentice Hall

[17] Ugural, A. C. (1999). "*Stresses in plates and shells*" (2nd Ed.). Singapore: McGraw-hill.

Ventsel, E and Krauthamer, T., (2001). "*Thin Plates and Shells: Theory, Analysis and Application*". Marcel Dekker, New York.

[18] www.brightstorm.com>vibrations and wav