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Flexural Strength Of Optimized Granite Partially Substituted By Palm Kernel Shell Concrete Using Scheffe'S Theory

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Abstract: *This work was carried out to develop mathematical model that can predict the response (modulus of rupture) of Granite-palm kernels shells aggregate concrete using Scheffe's method of optimization. Scheffe considered experiment with mixtures in which the desired property depends on the proportions of the constituent materials present as atoms of the mixture. A simplex lattice which can be described as a structural representation of lines joining the atoms of a mixture can be used as a mathematical space in model experiments involving mixtures by considering the atoms as the constituent components of the mixture. Concrete Beams were casted consisting of thirty (30) different mix ratios. The first fifteen (15) mix ratios were used to determine the coefficients of the model and the second fifteen (15) mix ratios were used to validate the model. The beams were cured for 28 days before testing. The values of responses (Results) determined from the models agreed with the corresponding values that were obtained from the experimental results. The formulated models were tested for adequacy using F – statistic test and were found to be adequate. The batch with the point (A1) , with mix ratio of 0.52:1:1.47:2.75:0.36 (water: cement: sand: Granite: palm kernel shells). 5% replacement of palm kernel shells has the highest Flexural strength of 3.8N/mm2, followed by point A2, A3 and A1,2 with modulus of rupture 3.4N/mm2, 2.9N/mm2 and 2.8N/mm2 respectively, The formulated models can predict all possible combinations of mix proportions if the value of modulus of rupture is given. Conversely, it can determine the modulus of rupture if a mix proportion is specified*

Keywords: *Optimization, Strength, Aggregates, Proportions, Models, Design and Samples*

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1. INTRODUCTION

Concrete is defined in Wikipedia as a composite material composed of aggregates bound together with a fluid cement that cures to a solid over time. in (student Encarta dictionary 2009) as a mixture of sand, cement, aggregate and water in specific proportions that hardens to a strong stony consistency over varying length of time. The aggregate in this context refers to rock particles of size above 5mm. American Concrete Institute also sees concrete as an engineering material made from a mixture of Portland cement, water, fine and coarse aggregate and small amount of air.. Concrete is an artificial material similar in appearance and properties to some natural lime stone rock. It is a man-made composite, the major constituent being natural aggregate such as gravel, or crushed rock, sand and fine particles of cement powder all mixed with water. The concrete as time goes on through a process of hydration of the cement paste, producing a required strength to withstand the load [7]. [11] defines concrete as a composite material consisting of a binding medium within which the particles are embedded. To achieve this composite material from the constituents, we need to follow all laid down rules, design standard and codes of practice. These methods took care of the shortcoming of the historical methods. Despite all of these advantages, the methods cannot be used to achieve an efficiently optimized mixture for a given criterion. Also, they require trial mixes . Thus, in 1958 Scheffe developed the simplex design method, which uses the theory of statistics and experiments to obtain models that can be used to determine mix proportions for a specified criterion [13]. The advantages of this method are that there is no need for trial mixes, saves time and cost, and can be used with computers to determine concrete mixes for a specified criterion. Researchers have applied Scheffe's simplex design method of optimisation and they came up with interesting results.

[6] formulated models for prediction of compressive strength of sandcrete blocks using Scheffe's and Osadebe's optimization theories. The results of the predictions were comparatively analysed and it was found that the two models are acceptable.

[12] developed models for optimisation of compressive and flexural strength of mound soil concrete using Scheffe's method.

[10] worked on concrete mixture design and generated a model for optimisation of concrete cube strength using Scheffe's optimisation theory. Statistical tools which were used to test the adequacy of the model agreed to the acceptance of same.

[9] developed a model for optimisation of strength of palm kernel shell aggregate concrete using Scheffe's simplex theory.

This work evaluates the modulus of rupture of concrete beam specimens made from Granite – Palm kernel shell aggregate concrete. This work also incorporates the formulation of mathematical models using Scheffe's simplex lattice design technique for the determination of flexural strength of concrete. The formulated models would predict the flexural strength of concrete made from Granite-Palm kernel shell aggregate of similar characteristics. Modulus of rupture, which is a measure of flexural strength, is the property of a solid that indicates its ability to resist bending. It is one of the basic parameters for computing deflection in reinforced concrete structures [1]. The design of some structures like dams (under earthquake conditions), concrete pavement (such as highway) and airfield pavements, is often based on the modulus of rupture of concrete. Such structures are required to resist tensile stress from two main sources namely wheel loads and volume changes [14]. Wheel loads may cause high tensile stress due to bending if there is an inadequate subgrade support. Volume changes as a result of variations in temperature and moisture may produce tensile stress due to warping and the movement of the slab along the subgrade. It is therefore necessary to assess the modulus of rupture of the concrete either from compressive strength or independently. Although concrete is not normally designed to resist direct tension, the knowledge of tensile strength is necessary in estimating the load under which cracking will occur [8].

II. MATERIALS AND METHODS

A. MATERIALS

Materials used in this research work are; Palm kernel shells, granite, portland cement and sharp sand.

Palm Kernel Shells: Palm Kernel shells were sourced from Ndenmili Community in Ndokwa west Local Government Area, Delta State, Nigeria. The palm kernel shells were washed to remove impurities that may have effect and sun dried.

Fine Aggregate (Sharp Sand). The sand for this project is river sand popularly known as sharp sand. It was obtained locally from open mark*et* along Ugbowo Lagos express road Benin City, Edo state.

Granite: Granite ranges in size from 5mm to 10mm were used for this research work. The granite was obtained from a building material market in Benin City, Edo State, Nigeria.

Portland Cement: the cement used for this work was ordinary Portland cement from open mark*et al*ong Ugbowo Lagos express road Benin City, Edo state.

Water: The water for the production of the concrete was obtained from the school (University of Benin) pipe burn water in the laboratory and it was fit for drinking and production of concrete.

2.2 Methods

The concrete specimens for flexural strength test were prepared in accordance with BS EN 12390-1 (2021). Three Concrete beams were cast in 150mm x 150mm x 600 mm moulds from each concrete mix ratio totaling Ninety beam specimens. The flexural strength test was done as specified by BS EN 12390-2 (2021). After 28 days from the day of production of concrete specimens, the cured samples were subjected to flexure strength using a symmetrical two-point loading flexural machine to the point of failure. The modulus of rupture is calculated using Equation. (1).

$$\delta f = \frac{PL}{bd^2} \quad (1)$$

where, P = Maximum load, L = The distance between supporting rollers, b and d are the lateral dimensions of the beam

In this work, Henry Scheffe's Simplex lattice design was used to formulate mathematical models which would be used to predict possible combinations of concrete components that will produce a specified modulus of rupture and vice versa.

Thus, if a mixture has a total of q components and Xi (as given in Equation. (2) be the proportions of the components (ingredients) of the ith component in the mixture such that

$$Xi \geq 0 \; (i = 1,2, \ldots \ldots q) \quad (2)$$

$Xi \geq 0$ is the components proportion

and assuming the mixture to be a unit quantity, then the sum of all the proportions of the component must be equal unit as provided in Equation (3). That is;

$$X_1 + X_2 + X_3 + \cdots + X_{q-1} + X_q = 1 \quad (3)$$

2.2.1 Scheffe's Factor Space

Mix components are assumed to interact within a factor space. The concrete used in this work is a 5-component mixture, consist of water, cement, sand granite and palm kernel shell, the number of components q is equals five, the factor space used for the analysis is q-1, that is four dimensional factor space which was analyzed using a pentahedron having a four-dimensional factor space illustrated in Fig1

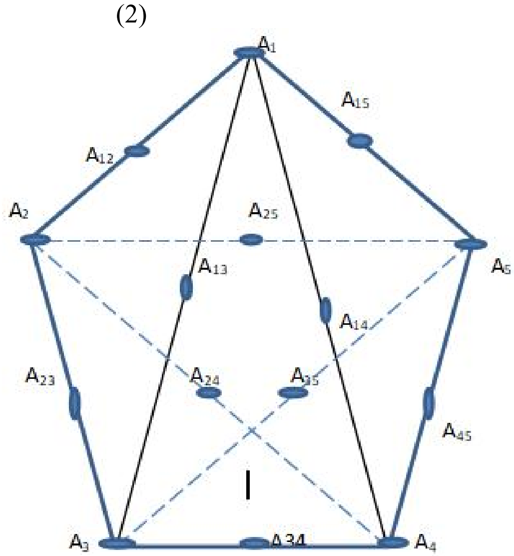


Fig.1: Five Components Mixture in A Four Dimensional Factor Space Showing 15 Points of Observation

2.2.2 Actual and Pseudo Components

The pseudo components represent the proportions of the ith component in the concrete mixture. At any point in the factor space, the summation of the pseudo components must be equal to one. No pseudo component is more than one or less than zero as represented in Equation. (4).

$$0 \leq Xi \leq 1 \quad (4)$$

The number (N) required for the mixture experiment is given by Scheffe, H. (1958) as;

$$N = \frac{(q+m-1)!}{m!(q-1)!} \quad (5)$$

N = Number of mix required

m = Degree of polynomial equation

q = Number of components in the mixture

q = 5,

m = 2

$$N = \frac{(5+2-1)!}{2!(5-1)!}$$

$$= \frac{6!}{2!\times 4!} = \frac{6\times5\times4\times3\times2\times1}{2\times1\times4\times3\times2\times1} = \frac{720}{8} = 1$$

5

15 is the number of mixture required in this experiment.

The coefficients in terms of pseudo components, xi and laboratory response of the first fifteen ratios gave the following relations as regression model equation of H. Scheffe's (5,2) simplex design [4].

$$Y(x) = A_1 X_1 (2X_1 - 1) + A_2 X_2 (2X_2 - 1) + A_3 X_3 (2X_3 - 1) + A_4 X_4 (2X_4 - 1) + A_5 X_5 (2X_5 - 1) + 4A_{12} X_1 X_2 + 4A_{13} X_1 X_3 + 4A_{14} X_1 X_4 + 4A_{15} X_1 X_5 + 4A_{23} X_2 X_3 + 4A_{24} X_2 X_4 + 4A_{25} X_2 X_5 + 4A_{34} X_3 X_4 + 4A_{35} X_3 X_5 + 4A_{45} X_4 X_5 \quad (6)$$

Equation (6) is the response function for optimization of Granite Palm Kernel shells aggregate concrete consisting of five components. Ai and Aij are the response (compressive strengths and flexural strength) at the point i and ij. The values of these responses are determined by carrying out flexural test on the beams using Granite – palm kernel shells aggregate concrete.

2.2.3 Equation for Actual and Pseudo Component Interaction

[10] and [5] provide an equation of Scheffe's elucidation of the relationship between the pseudo component and the actual component in their mixture designs. From Equations 7-10, the actual components of the mix design can be derived from the pseudo components and vice versa

Let the pseudo component be X and actual component be Z.

$$[Z] = [A][X] \quad (7)$$

Where [A] is the matrix of coefficients

$$[X] = [Z][A]^{-1} \quad (8)$$

let,

$$[A]^{-1} = [B] \quad (9)$$

Therefore,

$$[X] = [B][Z] \quad (10)$$

The actual components [Z] of the fiver-component mixture are determined by multiplying the values of matrix [A] with values of matrix [X] (pseudo components) as shown in Equation 8, the result of which is given in Equation [11]

A_1=[0.49, 1, 1.33, 2.7, 0.14]

A_2=[0.55, 1, 1.6, 2.81, 0.31]

A_3=[0.58, 1, 1.72, 2.79, 0.49]

A_4=[0.62, 1, 1.92, 2.84, 0.71]

A_5=[0.68, 1, 2.25, 2.87, 0.96]

First run

$$\begin{vmatrix} 0.49 \\ 1 \\ 1.33 \end{vmatrix} = \begin{vmatrix} X_{1,1} & X_{1,2} & X_{1,3} & X_{1,4} & X_{1,5} \\ X_{2,1} & X_{2,2} & X_{2,3} & X_{2,4} & X_{2,5} \\ X_{3,1} & X_{3,2} & X_{3,3} & X_{3,4} & X_{3,5} \end{vmatrix} \times \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} 2.7 \\ 0.14 \end{vmatrix} \quad \begin{vmatrix} X_{4,1} & X_{4,2} & X_{4,3} & X_{4,4} & X_{4,5} \\ X_{15} & X_{52} & X_{53} & X_{54} & X_{55} \end{vmatrix} \quad \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$X_{1,1} = 0.49 \quad X_{2,1} = 1 \quad X_{3,1} = 1.33 \quad X_{4,1} = 2.7 \quad X_{5,1} = 0.14$

Second run

$X_{1,2} = 0.55 \quad X_{2,2} = 1 \quad X_{3,2} = 1.61 \quad X_{4,2} = 2.81 \quad X_{5,2} = 0.31$

Third run

$X_{1,3} = 0.58 \quad X_{2,3} = 1 \quad X_{3,3} = 1.77 \quad X_{4,3} = 2.79 \quad X_{5,3} = 0.49$

Fourth run

$\mathbf{X_{1,4} = 0.62 \quad X_{2,4} = 1 \quad X_{3,4} = 1.92 \quad X_{4,4} = 2.84 \quad X_{5,4} = 0.71}$

Fifth run

$X_{1,5} = 0.68 \quad X_{2,5} = 1 \quad X_{3,5} = 2.25 \quad X_{4,5} = 2.87 \quad X_{5,5} = 0.96$

$$\begin{vmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{vmatrix} = \begin{vmatrix} 0.49 & 0.55 & 0.58 & 0.62 & 0.68 \\ 1 & 1 & 1 & 1 & 1 \\ 1.33 & 1.61 & 1.77 & 1.92 & 2.25 \\ 2.7 & 2.81 & 2.79 & 2.84 & 2.87 \\ 0.14 & 0.31 & 0.49 & 0.71 & 0.96 \end{vmatrix} X \begin{vmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{vmatrix} \qquad (11)$$

Five mix ratios (real and pseudo)that fine the vertices of pentahedron simplex lattice are shown in table 1.

Table 1: First Five Mix Ratio (Actual And Pseudo) Obtained From Scheffe's (5,2) Factor Space

Points	Real Mix Ratios					Pseudo Mix Ratios				
	Water	Cement	Sand	Granit	PKS	Water	Cement	Sand	Granit	PKS
	Z_1	Z_2	Z_3	Z_4	Z_5	X_1	X_2	X_3	X_4	X_5
A_1	0.49	1	1.33	2.7	0.14	1	0	0	0	0
A_2	0.55	1	1.6	2.81	0.31	0	1	0	0	0
A_3	0.58	1	1.72	2.79	0.49	0	0	1	0	0
A_4	0.62	1	1.92	2.84	0.71	0	0	0	1	0
A_5	0.68	1	2.25	2.87	0.96	0	0	0	0	1

For a (5, 2) simplex design, ten (10) other observation are needed to get a total of 15 observations needed for the development of the flexural strength equation. The remaining ten (10) points are located at the mid points of the lines joining the five (5) vertices. On successive substitution on these ten (10) pseudo mix ratios into Equation (6), the real mix ratios corresponding to the pseudo ones were obtained. Their values are show in Table 2.

Table 2: Additional Ten Mix Ratios (Real And Pseudo) For Formulation Of The Optimization Function.

Points	Real Mix Ratios					Pseudo Mix Ratios				
	Water	Cement	Sand	Granit	PKS	Water	Cement	Sand	Granit	PKS
	Z_1	Z_2	Z_3	Z_4	Z_5	X_1	X_2	X_3	X_4	X_5
$A_{1,2}$	0.52	1	1.47	2.755	0.36	0.5	0.5	0	0	0
$A_{1,3}$	0.535	1	1.55	2.745	0.45	0.5	0	0.5	0	0
$A_{1,4}$	0.555	1	1.63	2.77	0.425	0.5	0	0	0.5	0
$A_{1,5}$	0.585	1	1.79	2.785	0.55	0.5	0	0	0	0.5
$A_{2,3}$	0.565	1	1.69	2.8	0.4	0	0.5	0.5	0	0
$A_{2,4}$	0.585	1	1.77	2.825	0.51	0	0.5	0	0.5	0
$A_{2,5}$	0.615	1	1.93	2.84	0.635	0	0.5	0	0	0.5
$A_{3,4}$	0.6	1	1.85	2.815	0.6	0	0	0.5	0.5	0
$A_{3,5}$	0.63	1	2.01	2.83	0.725	0	0	0.5	0	0.5
$A_{4,5}$	0.65	1	2.09	2.855	0.835	0	0	0	0.5	0.5

2.2.4 Control Points

Fifteen control points used are C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, $C_{1,1}$, $C_{1,2}$, $C_{1,3}$, $C_{1,4}$, and $C_{1,5}$. The actual components and the corresponding pseudo components are shown in table 3.

Table 3: Actual And Pseudo Components Of Fifteen Control Points Observation.

Points	Real Mix Ratios					Pseudo Mix Ratios				
	Water	Cement	Sand	Granit	PKS	Water	Cement	Sand	Granit	PKS
	Z_1	Z_2	Z_3	Z_4	Z_5	X_1	X_2	X_3	X_4	X_5
C_1	0.56	1	1.66	2.785	0.48	0.25	0.25	0.25	0.25	0
C_2	0.575	1	1.74	2.793	0.543	0.25	0.25	0.25	0	0.25
C_3	0.585	1	1.78	2.805	0.598	0.25	0.25	0	0.25	0.25
C_4	0.593	1	1.82	2.8	0.643	0.25	0	0.25	0.25	0.25
C_5	0.608	1	1.89	2.828	0.618	0	0.25	0.25	0.25	0.25
C_6	0.584	1	1.78	2.802	0.576	0.2	0.2	0.2	0.2	0.2
C_7	0.548	1	1.61	2.774	0.434	0.3	0.3	0.3	0.1	0
C_8	0.554	1	1.64	2.777	0.459	0.3	0.3	0.3	0	0.1
C_9	0.566	1	1.68	2.792	0.525	0.3	0.3	0	0.3	0.1
$C_{1,0}$	0.575	1	1.73	2.786	0.579	0.3	0	0.3	0.3	0.1
$C_{1,1}$	0.593	1	1.82	2.815	0.549	0	0.3	0.3	0.3	0.1
$C_{1,2}$	0.613	1	1.92	2.82	0.689	0.1	0	0.3	0.3	0.3
$C_{1,3}$	0.604	1	1.87	2.826	0.635	0.1	0.3	0	0.3	0.3
$C_{1,4}$	0.592	1	1.82	2.811	0.569	0.1	0.3	0.3	0	0.3
$C_{1,5}$	0.574	1	1.72	2.802	0.494	0.1	0.3	0.3	0.3	0

Table 4. Mix Proportions (Actual Components) For The Concrete Beam.

Points	Real Mix Ratios					Pseudo Mix Ratios				
						Water	Cement	Sand	Granit	PKS
	Z_1	Z_2	Z_3	Z_4	Z_5	(Kg)	(Kg)	(Kg)	(Kg)	(Kg)
A_{12}	0.49	1	1.33	2.7	0.14	207	423	567	2262	70
A_{13}	0.55	1	1.6	2.81	0.31	210	383	617	2128	140
A_{14}	0.58	1	1.72	2.79	0.49	212	365	632	2014	211
A_{15}	0.62	1	1.92	2.84	0.71	210	338	654	1902	284
A_{23}	0.68	1	2.25	2.87	0.96	210	309	700	1756	350
A_{24}	0.52	1	1.47	2.755	0.36	204	392	580	2137	167
A_{25}	0.535	1	1.55	2.745	0.45	204	381	595	2070	202
A_{34}	0.555	1	1.63	2.77	0.425	209	376	615	2062	189
A_{35}	0.585	1	1.79	2.785	0.55	209	357	644	1970	232
A_{45}	0.565	1	1.69	2.8	0.4	210	372	632	2059	175
A_{12}	0.585	1	1.77	2.825	0.51	210	359	638	2006	216
A_{13}	0.615	1	1.93	2.84	0.635	210	342	664	1921	256
A_{14}	0.6	1	1.85	2.815	0.6	210	350	650	1948	248
A_{15}	0.63	1	2.01	2.83	0.725	210	333	675	1867	285
A_{23}	0.65	1	2.09	2.855	0.835	210	323	678	1826	318
C_1	0.56	1	1.66	2.785	0.48	207	370	617	2037	209
C_2	0.575	1	1.74	2.793	0.543	207	360	631	1991	231
C_3	0.585	1	1.78	2.805	0.598	207	354	634	1966	250
C_4	0.593	1	1.82	2.8	0.643	207	350	640	1938	265
C_5	0.608	1	1.89	2.828	0.618	210	346	657	1934	252
C_6	0.584	1	1.78	2.802	0.576	208	356	636	1972	242
C_7	0.548	1	1.61	2.774	0.434	206	377	609	2067	193

Table 4. Mix Proportions (Actual Components) for the Concrete Beam and Cube Cont'd

C_8	0.554	1	1.64	2.777	0.459	206	373	615	2048	202
C_9	0.566	1	1.68	2.792	0.525	206	365	618	2016	226
$C_{1,0}$	0.575	1	1.73	2.786	0.579	206	359	626	1980	245
$C_{1,1}$	0.593	1	1.82	2.815	0.549	210	354	647	1974	230
$C_{1,2}$	0.613	1	1.92	2.82	0.689	209	341	657	1902	277
$C_{1,3}$	0.604	1	1.87	2.826	0.635	209	346	650	1935	259
$C_{1,4}$	0.592	1	1.82	2.811	0.569	209	353	647	1963	237
$C_{1,5}$	0.574	1	1.72	2.802	0.494	209	364	631	2017	212

III. RESULTS AN DISCUSSION

3.1 Flexural Strength Test

Flexural strength test was performed to estimate the tensile load at which concrete may crack. The theoretical maximum tensile stress reached in the bottom fiber of the test beam is known as modulus of rupture. Modulus of rupture is a function of bending resistance of a concrete beam section. The design of dams (under earthquake conditions) and concrete slabs such as highway and airfield pavements are often based on the flexural strength of concrete because the structures are significantly subject to bending in service.

The Modulus of rupture (flexural strength), of granite – palm kernel shells aggregate concrete decrease with increase in percentage replacement of granite with palm kernel shells as shown in Table 5. As quantity of palm kernel shells increases, the specific area increases, this requires more cement paste to bond properly with the kernel shells. Strength requires good bonding of the aggregates and cement. This is in agreement in the work of (Daniel *et al.* 2012;Raheem et al., 2008 and Jurnaat et al., 2009) Thus, as bonding reduces with increase in replacement of palm kernel shell, flexural strength reduces.

The cured samples were subjected to tension using a symmetrical two-point loading flexural machine to the point of failure, in order to determine the flexural strength of the structural members, needed to formulate and validate the optimization function. Granite – Palm Kernel shells aggregate concrete specimens' beams with size (150 X 150 X 160) mm^3 were casted. Total number of ninety beams was casted for the thirty (30) mix ratios in Table 4. Three beams from each were produced. The first 45 element made from Table 3.4 were used for the formulation of the optimization model, the second set of 45 elements were used to validate the optimization model.

The average strength values range between 1.42N/mm^2 and 3.85N/mm^2, many mixes fall between 1.7-2.9N/mm^2, showing moderate performance. Some mixes show very close flexural values, e.g. point A_1, some show wider variation. Only point A_1 meets required level for standard structural concrete of range 3.5-4.0N/mm^2 at 28 days, (BS EN 1992-1-1 2004). The others fall below, meaning they might not be suitable for heavy structural applications but could be used for lightweight concrete or non-structural purposes.

3.2 Optimization Function for predicting the flexural Strength of the Concrete

Results of the flexural strength from the main experiments and control experiment are given in tables 5

Table 5: Mean Modulus of Rupture, Laboratory Test Results of Concrete Beams at 28 days of Granite – Palm Kernel Shells Aggregate Concrete

POINTS	BEAM 1 (N/mm^2)	BEAM 2 (N/mm^2)	BEAM 3 (N/mm^2)	TOTAL (N/mm^2)	AVERAGE (N/mm^2)	MODULUS OF RUPTURE (N/mm^2)
A_1	32	32.9	32.6	97.5	32.5	3.851851852
A_2	29	27.2	29.3	85.5	28.5	3.377777778
A_3	23.8	24	27.2	75	25	2.962962963
A_4	19.7	18.2	19.1	57	19	2.251851852
A_5	16	14.5	14.5	45	15	1.777777778
$A_{1,2}$	22	23.4	26.6	72	24	2.844444444
$A_{1,3}$	19.2	18	21.3	58.5	19.5	2.311111111
$A_{1,4}$	21.1	19	21.4	61.5	20.5	2.42962963
$A_{1,5}$	17.9	19.1	20	57	19	2.251851852
$A_{2,3}$	17	18.1	17.4	52.5	17.5	2.074074074
$A_{2,4}$	14.9	14.9	15.2	45	15	1.777777778
$A_{2,5}$	14.3	13.5	14.2	42	14	1.659259259
$A_{3,4}$	18.6	19.1	19.3	57	19	2.251851852
$A_{3,5}$	10.8	11	9.7	31.5	10.5	1.244444444
$A_{4,5}$	13	10.9	12.1	36	12	1.422222222

Table 5: Mean Modulus of Rupture, Laboratory Test Results of Concrete Beams at 28 days of Granite – Palm Kernel Shells Aggregate Concrete Cont'd

POINTS	BEAM 1 (N/mm^2)	BEAM 2 (N/mm^2)	BEAM 3 (N/mm^2)	TOTAL (N/mm^2)	AVERAGE (N/mm^2)	MODULUS OF RUPTURE (N/mm^2)
CONTROL						
C_1	15	17.7	15.3	48	16	1.896296296
C_2	16.7	17	18.8	52.5	17.5	2.074074074
C_3	15.3	16	15.2	46.5	15.5	1.837037037

C_4	10.6	11	11.4	33	11	1.303703704
C_5	18.9	17.6	20.5	57	19	2.251851852
C_6	13.8	14	12.7	40.5	13.5	1.6
C_7	12.2	11.9	11.9	36	12	1.422222222
C_8	14.6	13	12.9	40.5	13.5	1.6
C_9	13.9	14.1	15.5	43.5	14.5	1.718518519
C_{10}	17.3	17	16.7	51	17	2.014814815
C_{11}	10.4	9.2	10.4	30	10	1.185185185
C_{12}	12	10	11	33	11	1.303703704
C_{13}	11.6	10.1	11.3	33	11	1.303703704
C_{14}	12.1	11	11.4	34.5	11.5	1.362962963
C_{15}	23.9	24.8	26.3	75	25	2.962962963

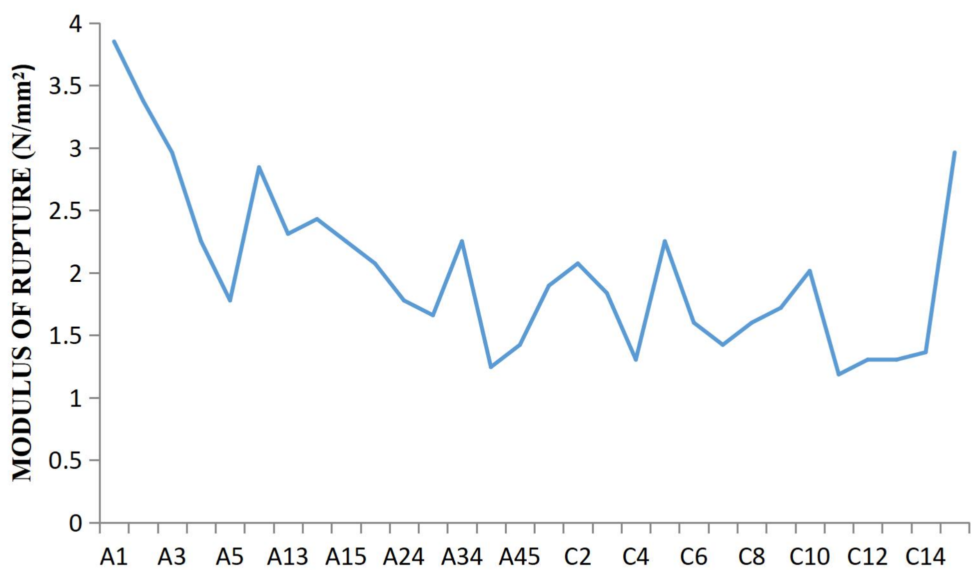


Chart of Mean Modulus of Rupture, Laboratory Test Results of Concrete Beams at 28 days of Granite – Palm Kernel Shells Aggregate Concr…

From Table 5, the coefficients of the equation (6) are obtained as follows;

$A_1 = 3.85$

$A_2 = 3.38$

$A_3 = 2.96$

$A_4 = 2.25$

$A_5 = 1.78$

$A_{12} = 4(2.74) = 11.36$

$A_{13} = 4(2.31) = 9.24$

$A_{14} = 4(2.43) = 9.72$

$A_{15} = 4(2.25) = 9.0$

$A_{23} = 4(2.07) = 8.28$

$A_{24} = 4(1.78) = 7.12$

$A_{25} = 4(1.66) = 6.64$

$A_{34} = 4(2.25) = 9.0$

$A_{35} = 4(1.24) = 4.96$

$A_{45} = 4(1.42) = 5.68$

Putting the above obtained values of coefficients into Equation (6) above gives;

$$Y(x) = 3.85X_1(2X_1-1) + 3.38X_2(2X_2-1) + 2.96X_3(2X_3-1) + 2.25X_4(2X_4-1) + 1.78X_5(2X_5-1)$$
$$+ 11.36X_1X_2 + 9.24X_1X_3 + 9.72X_1X_4 + 9.0X_1X_5 + 8.28X_2X_3 + 7.12X_2X_4 + 6.64X_2X_5$$
$$+ 9.02X_3X_4 + 4.96X_3X_5 + 5.68X_4X_5 \quad (12)$$

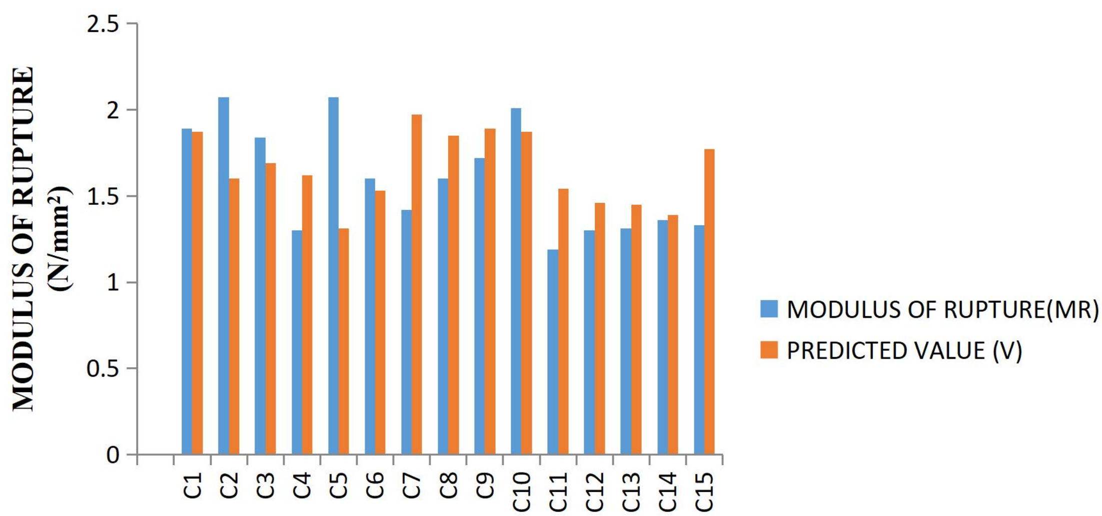
Equation (4.2) is the mathematical model for predicting and optimization of the Modulus of rupture of Granite palm kernel shells aggregate concrete based on 28 days strength.

Substituting mix ratios of points C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14} and C_{15}. into the equation (12) give the model response (Modulus of rupture) as in Table 6

Table 6: Model (Predicted) and Experimental Results of Modulus of Rupture of Granite – Palm Kernel Shells Aggregate Concrete

POINTS	BEAM 1 (N/mm^2)	BEAM 2 (N/mm^2)	BEAM 3 (N/mm^2)	TOTAL (N/mm^2)	AVERAGE (N/mm^2)	MODULUS OF RUPTURE(MR) (N/mm^2)	PREDICTED VALUE (V) (N/mm^2)
C_1	15	17.7	15.3	48	16	1.89	1.87
C_2	16.7	17	18.8	52.5	17.5	2.07	1.6
C_3	15.3	16	15.2	46.5	15.5	1.84	1.69
C_4	10.6	11	11.4	33	11	1.3	1.62
C_5	17.1	16.6	18.8	57	19	2.07	1.31
C_6	13.8	14	12.7	40.5	13.5	1.6	1.53
C_7	12.2	11.9	11.9	36	12	1.42	1.97

C_8	**14.6**	**13**	**12.9**	**40.5**	**13.5**	**1.6**	**1.85**
C_9	**13.9**	**14.1**	**15.5**	**43.5**	**14.5**	**1.72**	**1.89**
C_{10}	**17.3**	**17**	**16.7**	**51**	**17**	**2.01**	**1.87**
C_{11}	**10.4**	**9.2**	**10.4**	**30**	**10**	**1.19**	**1.54**
C_{12}	**12**	**10**	**11**	**33**	**11**	**1.3**	**1.46**
C_{13}	**11.6**	**10.1**	**11.3**	**33**	**11**	**1.31**	**1.45**
C_{14}	**12.1**	**11**	**11.4**	**34.5**	**11.5**	**1.36**	**1.39**
C_{15}	**11.2**	**10.8**	**11.6**	**33.6**	**11.2**	**1.33**	**1.77**



Bar Chart of Model (Predicted) and Experimental Results of Modulus of Rupture of Granite – Palm Kernel Shells Aggregate Concrete

3.3 Test for Adequacy of the Mathematical Models

Fisher's statistical tool was used to compare the predicted control values of Modulus of rupture which were not involved in the formulation of the model equations and control results from the experiment.

The adequacy of the mathematical model was done using fisher test at 95% confidence level on the Modulus of rupture at the control points C_1, C_2, C_3, C_4, C_5, C_6 ,C_7, C_8, C_9 ,C_{10} ,C_{11}, C_{12}, C_{13}, C_{14} and C_{15}.

Two hypothesis were established, which are;

1. Null Hypothesis There is no significant difference between the model's results and experimental results.

2. Alternative Hypothesis There is a significant difference between the model's results and experimental results. The assumption of 95% accuracy implies that 5% or below of the model's results will be incorrect.

The Null Hypothesis will be accepted if:

$$\frac{1}{F} < \frac{S_i^2}{S_t^2} < F$$

Provided by Fisher must be satisfied for the developed model equation to be considered adequate. Where F is Fisher value at 5% significance level or 95% confidence level, Si^2 is the larger value between SV^2 and $S(Mr)^2$ (St^2 is the smaller value between $S(Mr)^2$ and SV^2).

If the condition is not satisfied then the alternative Hypothesis will be accepted. The test was carried out using the table 7 below.

Table 7: Fisher-Statistical Test Computation for the Model

POINTS	MODULUS OF RUPTURE(MR)	PREDICTED VALUE (V)	MR-I
C_1	1.89	1.87	0.29
C_2	2.07	1.6	0.47
C_3	1.84	1.69	0.24
C_4	1.3	1.62	-0.3
C_5	2.07	1.31	0.47
C_6	1.6	1.53	0
C_7	1.42	1.97	-0.18
C_8	1.6	1.85	0
C_9	1.72	1.89	0.12
C_{10}	2.01	1.87	0.41
C_{11}	1.19	1.54	-0.41
C_{12}	1.3	1.46	-0.3
C_{13}	1.31	1.45	-0.29
C_{14}	1.36	1.39	-0.24
C_{15}	1.33	1.77	-0.27
TOTAL	24.01	24.81	
MEAN(M)	1.60	1.65	

MR = Modulus of rupture from control experiment.

V = Modulus of rupture from second degree polynomial equation.

N = 15

$$S(Mr)^2 = \frac{(Mr - Mean)^2}{N-1} = \frac{1.36}{14} = 0.097$$

$$SV^2 = \frac{(V - Mean)^2}{N-1} = \frac{0.602}{14} = 0.043$$

$$F_{Calculated} = \frac{S_i^2}{S_t^2}$$

Where S_i^2 is the greater of $S(Mr)^2$ and Sv^2 , while St^2 is the smaller of the two.

Here,

$S_i^2 = S(Mr)^2 = 0.097$ and $S_t^2 = SV^2 = 0.043$

$$F_{Calculated} = \frac{0.097}{0.043} = 2.26$$

The mathematical model is acceptable at 95% confidence level if;

$$\frac{1}{F\alpha(v_1, v_2)} < \frac{S_i^2}{S_t^2} < F$$

Significant level α = 1-0.95 = 0.05;

Degree of Freedom, v = N – 1 = 14

Therefore, F α (v_1,v_2) = F0.05 (14, 14)

From the standard F – Statistical Table, F0.05 (14, 14) = 2.443

$$\frac{1}{F\alpha(v_1, v_2)} = \frac{1}{2.26} = 0.44$$

Hence, the condition that;

$$\frac{1}{F\alpha(v_1, v_2)} < \frac{S_i^2}{S_t^2} < F$$

Which is 0.44<2.26< 2.443 is satisfied

Therefore the Null Hypothesis that, "there is no significant difference between the experimental and the model expected results is accepted. This implies that the mathematical model equation is adequate.

IV. CONCLUSION AND RECOMMENDATIONS

This research work has the following conclusions;

a) The batch with the point (A_1) with 5% replacement of palm kernel shells has the highest Flexural strength of 3.8N/mm^2, followed by point $A_{1,2}$ and $A_{2,3}$ with modulus of rupture 2.8N/mm^2 obtained from mix point ($A_{1,2}$), with mix ratio of 0.52:1:1.47:2.75:0.36 (water: cement: sand: Granite: palm kernel shells). The higher the percentage of Palm Kernel Shell in the mix design, the higher the volume of concrete produced per mix which gives high value of economy.

b) The use of Henry Scheffe's (5,2) polynomial equation, the mathematical model for the mix design of a five component of Granite-palm kernel shells aggregate concrete was formulated. Predictions were tested at 95% accuracy with the use of Fisher Test and found to be adequate and with known mix ratios, the mathematical model can predict the response (compressive strength/modulus of rupture) of Granite-palm kernels shells aggregate concrete

c) Following the result of the experiment conducted, it implies that Palm kernel shell concrete may be used for lightweight, non-structural or semi structural applications where reduced density and cost are prioritized over high strength.

1. REFERENCES

1. Anbuvelan, K. and Subramanian, K. (2014). "An empirical relationship between modulus of elasticity, modulus of rupture and compressive strength of M60 concrete containing metakaolin," Journal of Applied Science and Engineering Technology. 8 (11):1294–1298.
2. British Standards Institution, BS EN 12390-1 (2021). Testing hardened concrete - Part 1: Shape, dimensions and other requirements for specimens and moulds.
3. Daniel Y. Osei, E. and Nana J. (2012). "Experimental Study on Palm Kernel Shells as Coarse Aggregate in Concrete," International Journal of Science And Engineering Research, 3, 8 2229 - 5518.
4. Ezeh, J.C., Ibearugbulem, O.M., and Anya, U.C., (2010). Optimisation of aggregate composition of laterite/sand hollow block using Scheffe's simplex method. International Journal of Engineering, vol. 4(4), pp. 471-478.
5. Gamil, Y. M. R. and Bakar I. H. (2016). The Development of Mathematical Prediction Model to Predict Resilient Modulus for Natural Soil Stabilized by Pofa-OPC Additive for the Use in Unpaved Road Design, IOP conference Series Materials Science and Engineering, 136(1): 1-11.
6. Mama, B.O. and Osadebe, N.N. (2011). Comparative analysis of two mathematical models for prediction of compressive strength of sandcrete blocks using alluvial deposit, Nigerian Journal of Technology, vol. 30(3).
7. Maninder K. and K. Manpreet (2012). A Review on Utilization of Coconut Shell as Coarse Aggregates in Mass Concrete. International Journal of Applied Engineering Research, ISSN 0973-4562 Vol. 7 No.11.
8. Neville, A. M. (2012). "Properties of Concrete". Fifth Edition. Pearson. Harlow, England. The Concrete Centre website. [Online]. Available: https://www.concretecentre.com.
9. Obam, S.O., (1998). A model for optimisation of strength of palm kernel shell aggregate concrete. Unpublished thesis. Civil Engineering department, University of Nigeria, Nsukka.
10. Okere, C. E. (2013). Optimization of Modulus of Rupture of Concrete Using Scheffe's Simplex Theory. International Journal of Advanced Technology and Engineering Research,3: 1-6.
11. Olanipekun, E.A. (2006). A Comparative Study of Concrete Properties using Coconut Shell and Palm Kernel Shell as Coarse Aggregates. Journal of Building and Environment 41(13):297-301.
12. Orie, O. U., (2008). Five-component concrete mix optimisation using Scheffe's theory – A case study of mound soil as a mix component. Thesis (PhD). Civil Engineering Department, University of Benin.
13. Scheffe, H. (1958). Experiments with mixtures. Journal of Royal Statistical Society, Series B. Vol. 20, No 2, pp. 344-360.
14. Shetty, M. S. (2005). "Concrete Technology Theory and Practice,". Revised Edition. S. Chand and Company Ltd. New Delhi.