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Mathematical Modeling of Nature-Based Solutions for Enhanced Slope Stability and Erosion Control in Highway Embankments

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Abstract: *Highway embankments are crucial to transportation infrastructure but are challenged by increasing threats from climate change, including slope instability and erosion. This study directly contributes to sustainability by proposing a quantitative and integrative framework for incorporating Nature-Based Solutions (NbS) into highway embankment design—offering an environmental and green alternative to conventional engineering models. The study combines slope stability analysis (via the Morgenstern-Price method), seepage flow modelling (using Darcy's Law), and erosion prediction (through the Universal Soil Loss Equation, USLE) to evaluate the ecological and structural performance of vegetative coupled strategies. Key sustainability indicators such as improved Factor of Safety (FoS), reduced soil loss, and optimized hydraulic behaviourweremodelled to assess resilience under varying environmental conditions. Results indicates that NbS enhance long-term embankment stability and reduce erosion, with a critical slope length of 37.3 meters identified for intervention. By optimising the need for energy- and resource-intensive engineering interventions, and by utilizing the regenerative capacity of vegetation, this research reinforces sustainability by promoting eco-engineering, improving resilience to climate change, and supporting circular land-use practices in transportation infrastructure development.*
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Keywords: *Environmental Sustainability, Nature-Based Solutions (NbS); Slope Stability; Erosion Control; Mathematical Modeling; Highway Embankments*

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I. INTRODUCTION

The importance of highway pavements extends beyond their functional and practical benefits for road users; they are fundamental assets within transportation systems and critical components for societal functionality (Plati, 2019). Specifically, highway embankments play a crucial role in ensuring the stability and sustainability of road infrastructure, making their design and maintenance a priority. Pavement design, in the context of highway embankments, involves determining the appropriate structure and thickness of the pavement to be applied over varying soil formations within natural environmental conditions. This ensures a stable and durable riding surface capable of withstanding potential external factors (Qiu et al. 2024; Robinson et al., 2024; Wijaya et al., 2022).

To enhance the durability and performance of highway pavements, numerous institutions and research organisations have developed design methodologies incorporating empirical and mechanistic approaches. These methods model pavement performance and deterioration as a function of key variables, such as soil conditions, traffic loads, and environmental factors Yazdani, et al., 2024)..For flexible pavements, methods have traditionally focused on soil behaviour, theoretical frameworks, and statistical analysis of field and laboratory tests, which have allowed for a systematic classification and evaluation of pavement design approaches (Clarke et al.,2006; Emeka et al., 2018; Zakarka&Skuodis, 2023; Al-Dulaimi & Seyedi, 2023). Pavement design and performance are influenced by a complex interplay of variables, including pavement structure design, traffic loads, construction practices, maintenance strategies, and environmental conditions (Haas, 2001). For highway embankments, these factors are further complicated by slope stability and erosion challenges, which require integrated approaches to ensure long-term performance and sustainability. Addressing these challenges involves incorporating Nature-Based Solutions (NbS) into pavement and embankment design to mitigate environmental impacts, enhance resilience, and extend service life (Nikolaides, 2014; Mallick& El-Korchi, 2008, Nick, 2008; Mendoza-Sanchez, 2024; Kumar & Hayano, 2024; Dhakal et al., 2023; Świtała,, 2023; DiBiagio et al., 2024).

Climate change witnessed recently has resulted to focus on a number of related hydro-thermal environmental hazards (HTEHs) (Ota et al., 2023; Leinauer et al., 2024). Slope failure and erosion are two of the most common HTEHs experienced globally in sloping areas especially in highway pavements. The major contributors to these hazards include, changes in rainfall patterns, changes in wind and wave intensity and direction, changes in temperature patterns and climatic change resulting from negligence of taking care of nature and rapid urbanization. While conventional method of construction using concrete and/or steel structures have continued to be used as solution to mitigate the effects of slope failure and erosion (Jafari, N., & Puppala,2018;.Psarropoulos et al., 2024; Gall et al., 2024; Bathi et al., 2023;Dhanai,et al. 2022; Liu and Wang, 2024; Bračko et al.,2022; de Souza Batista et al.,2024)

However, recently there has been a drive for using Nature based solutions (NbS) for slope protection, erosion control and mitigating other engineering problems (e.g., Rodrigues et al., 2024; Pontee&Bassetti, 2024). NbS are typically deployed as a means of aligning with Mother Nature and fostering a sustainable ecosystem, utilizing nature's elements and principles to address diverse climate change and socio-environmental challenges such as water security, air and water pollution, extreme heat, biodiversity conservation, and human health .Blue-Green Infrastructure (BGI) encompasses natural and semi-natural areas, strategically planned, designed, executed and maintained alongside other environmental features (such as Nature-based Solutions, NbS) to provide a diverse range of ecosystem services(Ghosh et al.,2024 ; Paynter, 2024; Bridges et al., 2024; Prado et al., 2024; Gall et al., 2024; Bowyer, et al., 2024; Zedek, et al., 2024).

Despite the growing interest in Nature-Based Solutions (NbS) for slope stability and erosion control, there is a lack of integrated mathematical models that combine slope stability, seepage flow, and erosion control in a single framework. Existing studies often focus on individual aspects, neglecting the synergistic effects of vegetation, soil properties, and hydrological processes (Oorthuis, 2022).

This research targets to develop an integrated mathematical model that quantifies the contribution of Nature-Based Solutions (NbS) to sustainable slope stability and erosion control in highway embankments. By aligning with sustainable infrastructure principles, the research (1) incorporates vegetation reinforcement into slope stability analysis to reduce reliance oncarbon-laden structural solutions, (2) models seepage flow mobility using Darcy's law to simplify hydrological behaviour in eco-engineered slopes, and (3) integrates erosion control through the Universal Soil Loss Equation (USLE) (Tsige, 2019) to utilize soil conservation benefits. Generally, these components offer a science-based framework to guide the promotion of NbS as a climate-resilient and environmentally sustainable alternative in transportation infrastructure design and management.

II. METHODOLOGY

2.1 Mathematical Modeling

2.1.1 Theory

The integration of slope stability, seepage flow, and erosion control models is achieved through a step-by-step coupling of the equations. First, the Factor of Safety (FoS) is calculated using the Morgenstern-Price method, incorporating vegetation reinforcement. Next, seepage flow dynamics are modeled using Darcy's law, with pore water pressure directly influencing the FoS. Finally, soil erosion is quantified using the USLE, and its impact on soil cohesion is integrated into the FoS equation. This holistic approach allows for a comprehensive analysis of slope stability under the combined influence of vegetation, seepage, and erosion.

2.2 Slope Stability Model

The Factor of Safety (FoS) for slopes is modelled using the Morgenstern-Price method:

$$Fos = \frac{c' + (\sigma - u)tan\phi'}{\tau}$$

Where:

- c': Effective cohesion of the soil
- σ: Total normal stress
- u: Pore water pressure
- ϕ': Effective angle of internal friction
- τ: Shear stress along the failure surface

Vegetation reinforcement is incorporated as an additional shear strength component:

$$\Delta\tau_v = k_r R \qquad (2)$$

Where:

- k_r: Root reinforcement coefficient
- R: Root tensile strength

(Jiang et al., 2024)

2.3 Seepage Flow and hydraulic Modeling

Seepage flow through the slope is analyzed using **Darcy's law:**

$$q = k\frac{dh}{dx} \quad 3$$

Where:

q: Seepage rate
k: Hydraulic conductivity
$\frac{dh}{dx}$: Hydraulic gradient

Finite element methods (FEM) are employed to simulate water flow and pore pressure distribution, accounting for NbS such as drainage layers and vegetation -included evapotranspiration in line with the studies of Vishnu & Bharat (2021) and Sadeghi, et al. (2023).

2.4 Erosion Control Model

The **Universal Soil Loss Equation (USLE)** is adapted for vegetative cover:

$$A = R \cdot K \cdot L \cdot S \cdot C \cdot P \quad 4$$

Where:

A: Estimated soil loss
R: Rainfall erosivity
K: Soil erodibility
L: Slope length
S: Slope steepness
C: Cover management factor
P: Support practice factor

Vegetation coverage reduces C, effectively mitigation erosion (Li et al., 2017)

2.5 Slope Stability Model

The Factor of Safety (FoS) equation is:

$$FoS = \frac{c^{'} + (\sigma - u)tan\phi' + \Delta\tau_v}{\tau} \quad 5$$

Here, $\Delta\tau_v$=$k_rR$is the additional shear strength due to vegetation reinforcement. Substituting this into FoS equation:

$$FoS = \frac{c^{'} + (\sigma - u)tan\phi' + k_rR}{\tau} \quad 6$$

2.6 Incorporating Seepage Flow Dynamics

The pore water pressure u in the stability equation is directly influenced by seepage flow. Using Darcy's law:

$$u \propto q = k\frac{dh}{dx} \quad 7$$

Substituting $u = \propto k\frac{dh}{dx}$ (where $\propto$ is a proportionality constant representing the relationship between pore pressure and seepage rate):

$$FoS = \frac{c^{'} + \left(\sigma - \propto k\frac{dh}{dx}\right)tan\phi' + k_rR}{\tau} \quad 8$$

2.7 Incorporate Erosion Control

Soil erosion affects the stability of slopes by altering soil properties and reducing $c^{'}$. The soil loss from the USLE is:

$$A = R \cdot K \cdot L \cdot S \cdot C \cdot P \quad 9$$

To couple this with the stability model, we consider $c^{'} \propto 1 - \beta A$, where β is a factor indicating soil loss impacts cohesion. Substituting A into $c^{'}$:

$$c^{'} = c^{'}_0 \cdot (1 - \beta \cdot R \cdot K \cdot L \cdot S \cdot C \cdot P) \quad 10$$

Here $c^{'}_0$ is the initial soil cohesion before erosion. Substituting $c^{'}$ into the FoS equation:

$$FoS = \frac{c^{'}_0 \cdot (1 - \beta \cdot R \cdot K \cdot L \cdot S \cdot C \cdot P) + \left(\sigma - \propto k\frac{dh}{dx}\right)tan\phi' + k_rR}{\tau} \quad 11$$

2.8 Final Combined Equation

The holistic equation for slope stability, incorporating seepage flow and erosion control, becomes:

$$FoS = \frac{c^{'}_0 \cdot (1 - \beta \cdot R \cdot K \cdot L \cdot S \cdot C \cdot P) + \left(\sigma - \propto k\frac{dh}{dx}\right)tan\phi' + k_rR}{\tau} \quad 12$$

Where:

$c^{'}_0$: initial soil cohesion
β: Erosion impact coefficient

R, K, L, S, C, P: USLE parameters (rainfall, soil erodibility, slope length, steepness, cover management, and support practice factors)
σ: Total normal stress
$\propto$: Proportionality constant relating pore pressure to seepage rate
k: Hydraulic conductivity
$\frac{dh}{dx}$: Hydraulic conductivity
ϕ': Effective angle of internal friction
k_r: Root reinforcement coefficient
R: Root tensile strength
τ: Shear stress along the failure surface

This equation unifies the three modelling aspects into a single framework, allowing for a comprehensive analysis of slope stability under the influence of vegetation, seepage, and erosion.

2.8.1 Erosion Parameters:

c'_0: = 20KPa: initial soil cohesion
β = 0.015: Erosion impact coefficient$R =$
60: Rainfall erosivity index
$K = 0.4$: Soil erodibility index
L: Slope length (varies, e.g., 10 m to 50 m)
$S = 1.5$: Slope steepness factor
$C = 0.25$: Cover management factor
$P = 0.6$: Support practice factor
↓

2.8.2 Slope Stability Parameters:

σ = 90KPa: Total normal stress
ϕ = 28°: Effective internal friction angle
$K_r = 12\ kPa$: Root reinforcement coefficient
$R_{root} = 6kPa: Roottensilestrength$

2.8.3 Seepage Parameters:

$\propto\ = 0.6: Pore\ pressure\ proportionality\ constant$
$k = 0.02m/s$: Hydraulic conductivity
$$\frac{dh}{dx} = 0.1: Hydraulicgradient$$
Shear stress along the failure surface: **: T = 18kPa**

III. RESULTS AND DISCUSSION

3.1 Results

Table 1: Length Slope and the corresponding factor of safety

Slope Length (m)	Factor of Safety (Fos)
10	1.5
15	1.3
20	1.228571
25	1.157143
30	1.085714
35	1.014286
40	0.942857
45	0.871429
50	0.8

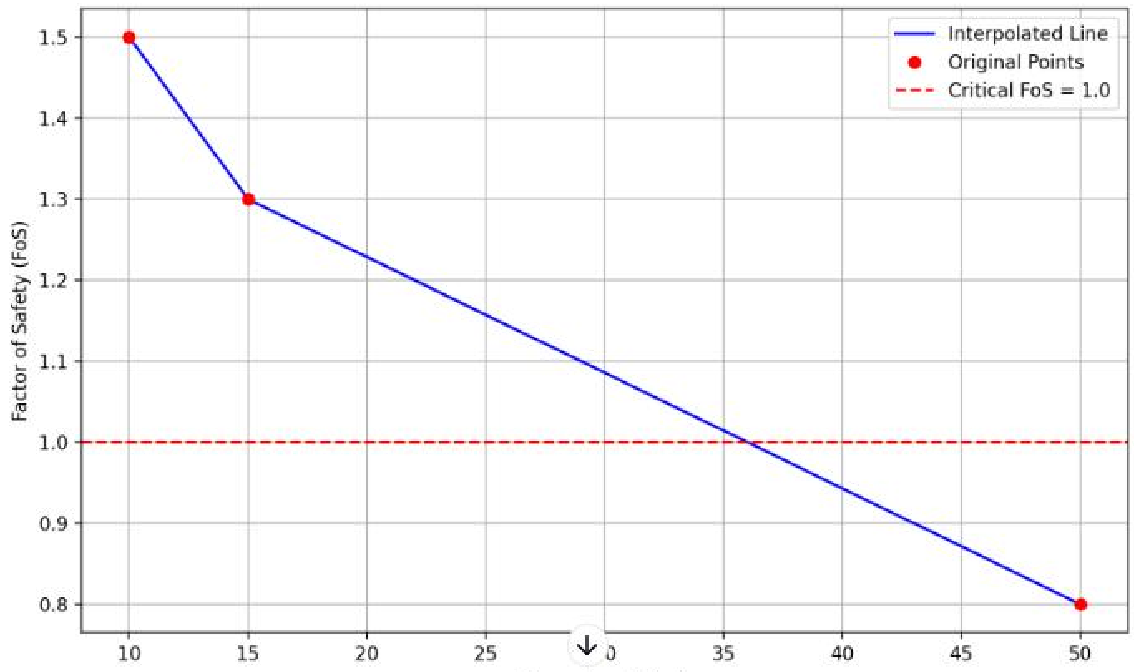

Figure 1: Factor of Safety Vs Slope Length

The relationship between the Factor of Safety (FoS) and slope length (L) is given by:

FoS= -0.0163L + 1.6079FoS
13

This indicates that the FoS decreases linearly with slope length at a rate of approximately 0.0163 per meter.
The slope becomes critically unstable (FoS=1.0) at approximately 37.3 meters.
The high R-squared value (0.973) confirms that the linear model fits the data well.

4. Discussions

The results affirm that Nature-Based Solutions (NbS) substantially enhance the sustainability of highway embankments by increasing the Factor of Safety (FoS) and reducing soil erosion. The evident linear relationship between FoS and slope length emphasizes that longer slopes are more prone to collapse, with a critical slope length of 37.3 meters identified. This has direct indications for sustainable infrastructure design, as it enables targeted, environmentally friendly interventions that reduces the use of energy-intensive structural methods. Integrating vegetation reinforcement into the slope stability models the long-term ecological and geotechnical benefits of NbS, highlighting there importance in promoting resilient and adaptive infrastructure while reducing dependency on conventional grey engineering techniques.

Our findings are consistent with previous research showing the effectiveness of vegetation in stabilizing slopes (e.g., Tardio and Mickovski, 2015; Fu et al., 2020). Moreover, the novelty of our work centres in the integrated modelling of vegetation reinforcement, seepage dynamics, and erosion control—offering a holistic and sustainability-oriented framework. This integrated method advances the state of knowledge by quantifying the synergistic environmental benefits of NbS, which is crucial for mainstreaming sustainable design practices in geotechnical engineering.

IV. CONCLUSION AND RECOMMENDATION

This study introduces a detailed mathematical modelling framework for determining the sustainability benefits of Nature-Based Solutions (NbS) in highway embankment design. By integrating slope stability analysis, seepage flow modelling, and erosion prediction, the research demonstrates that NbS significantly enhance structural resilience and environmental performance. The model's identification of a critical slope length (37.3 meters) supports targeted and resource-efficient applications of vegetation-based strategies, reducing both soil loss and infrastructure vulnerability. This research contributes to sustainable development by providing a science-backed, eco-friendly alternative to conventional engineering, aligning with global climate resilience and land restoration visions. Future work should explore the long-term performance of NbS across diverse climates and develop standardized design protocols to support widespread, sustainable implementation in transportation infrastructure.

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