NAU Journal of Civil Engineering (NAUJCVE) January/February 2026
Nnamdi Azikiwe University Journal of Civil Engineering (NAUJCVE)

Volume-5, Issue-1, pp--20-27
www.naujcve.com
Research Paper Open Access

Leveraging Probabilistic Modelling of Uncertainties to Mitigate Reinforce Concrete Structural Element Failure

[1]Uzoka Celestine Chukwudi and [2]Imarhiagbe Eghosa Akugbe

[12] Department of Civil Engineering, Igbinedion University, Okada, Edo State, Nigeria

Corresponding Author Email: uzokacelestine@yahoo.com

Abstract : *This study evaluates the structural safety and reliability of a reinforced concrete beam using both deterministic and probabilistic approaches. The probabilistic properties of material strengths and applied loads form the basis for an analytical mean-value assessment and Monte Carlo simulation (MCS). Flexural resistance and demand are computed using standard reinforced concrete beam theory, while structural safety is quantified through a limit state function. The probability of failure is estimated from fundamental probability principles. Beam geometric dimensions are treated as deterministic variables, as construction tolerances (±5–10 mm) are relatively small and well controlled on site compared with the inherent variability of material properties and loads.Results from the analytical mean-value approach indicate that the beam design is structurally safe, with the mean bending resistance exceeding the mean bending demand by a large margin, reflecting a conservative design. However, assessment of the constructed beam using mean values shows only a small reserve capacity, highlighting the limitations of deterministic analysis in capturing uncertainty. Monte Carlo simulation of the design beam demonstrates high reliability, with no failure cases recorded in all thirty simulations and consistently positive limit state values. In contrast, probabilistic analysis of the constructed beam reveals nine failure cases out of thirty simulations, corresponding to a probability of failure of approximately 0.30. These failures occur under unfavourable combinations of high load effects and reduced material strengths and reinforcement areas. The findings demonstrate that while mean-value analysis may classify a beam as safe, probabilistic modelling provides deeper insight into structural reliability and exposes potential risks associated with real construction variability.*

Keywords: *Probabilistic modelling; Monte Carlo simulation; Reinforced concrete beam; Structural reliability; Probability of failure*

Date of Submission: 05-01-2026 Date of acceptance: 11--01-2026

I. INTRODUCTION

Reinforced concrete (RC) structures form the backbone of global infrastructure because of their versatility, durability and relatively low construction cost (Neville, 2011). However, RC structural elements are inherently affected by uncertainties arising from material variability, construction deviations, environmental exposure and modelling simplifications, all of which influence structural reliability (Beck et al., 2013). Traditional deterministic design approaches assume single "characteristic values" for loads and resistances, but such simplified treatment often masks the true influence of uncertainties, resulting in either overly conservative or insufficiently safe structures (Melchers & Beck, 2018). Therefore, modern structural engineering increasingly adopts probabilistic modelling to quantify uncertainties and predict failure likelihood more realistically (Ang & Tang, 2007).

Probabilistic approaches allow explicit representation of uncertainty sources such as variability in concrete compressive strength, steel yield stress, reinforcement placement errors and unpredictable load fluctuations (Achenbach, 2019). They also enable propagation of these uncertainties using stochastic finite element models, reliability analysis methods and Monte-Carlo simulations to estimate failure probabilities for RC elements (Grubišić et al., 2019). These tools provide a more accurate basis for safety assessment and support performance-based design strategies where risks are numerically quantified (Ellingwood & Galambos, 1982).

In addition, deterioration processes—particularly corrosion of reinforcement—are highly uncertain and time-dependent. For example, chloride ingress rates, cover depth variability and threshold chloride concentration exhibit significant randomness, meaning corrosion initiation and propagation times must be treated probabilistically (Nogueira et al., 2012). Time-variant reliability approaches thus improve predictions of service life and guide decisions on inspection, maintenance and retrofitting (Seghier et al., 2021).

Probabilistic fragility analysis has also emerged as a powerful tool for assessing the seismic performance of RC members and buildings. By accounting for uncertainties in material strengths, hysteretic behaviour and seismic demand, fragility curves capture the likelihood of reaching various damage states during earthquakes (Blasi et al., 2023). Similar probabilistic frameworks have been applied to blast loading, impact actions and multi-hazard scenarios using surrogate models and machine-learning-based estimators (Yang et al., 2025; Zhang et al., 2023).

Given these advancements, leveraging probabilistic modelling to mitigate RC structural failures is essential for developing robust, risk-informed engineering solutions. This study synthesizes key probabilistic methods, characterizes uncertainty sources affecting RC behaviour and demonstrates how probabilistic outcomes support decision-making for failure mitigation. The next section provides a detailed background of the study.

Uncertainty in RC behaviour arises from material properties, geometric imperfections, construction tolerances, environmental deterioration and modelling assumptions (Achenbach, 2019). Concrete is highly heterogeneous, meaning its compressive strength, tensile capacity and elastic modulus vary spatially and between batches (Ghannoum et al., 2023). Reinforcement steel also exhibits scatter in yield stress, ultimate strength and bond properties (Lu et al., 1994). Geometric uncertainties—such as variation in cover depth and misplacement of bars—can significantly reduce member capacity and accelerate deterioration (Val et al., 2025).

Load uncertainties include variable live loads, wind actions, seismic excitations and accidental loads, all of which are characterized probabilistically (Ang & Tang, 2007). Environmental uncertainties such as chloride concentration, temperature variation and carbonation depth influence corrosion rates, making deterioration highly uncertain (Nogueira et al., 2012). Modelling uncertainties arise from simplified constitutive laws, boundary conditions and numerical approximations used in structural analyses (Achenbach, 2019).

A clear understanding of these uncertainties is essential for developing probabilistic limit-state functions for flexure, shear, bond failure, punching shear and deterioration-driven failures (Melchers & Beck, 2018).

A broad set of probabilistic tools exists to analyze uncertainties in RC structural performance. Monte-Carlo simulation (MCS) remains the most widely used technique because it makes minimal assumptions and can approximate failure probabilities accurately (Song & Kawai, 2023). However, direct MCS can be computationally demanding, especially for rare events or detailed nonlinear models. Therefore, advanced sampling methods such as importance sampling, subset simulation and Latin hypercube sampling are often used to reduce computational effort (Grubišić et al., 2019).

First- and second-order reliability methods (FORM/SORM) provide efficient approximations of failure probability by linearizing or quadraticising the limit-state function at the design point (Melchers & Beck, 2018). These methods are particularly useful in preliminary assessments or when computational budgets are limited.

Stochastic finite element methods (SFEM) have been developed to include spatial variability of material properties directly in structural models, providing improved accuracy in predicting crack patterns and failure loads (Ghannoum et al., 2023). Time-dependent reliability methods integrate deterioration models such as diffusion-based chloride ingress or Gamma-process corrosion models with reliability techniques to predict service life and failure evolution (Seghier et al., 2021).

Bayesian updating enables reduction of epistemic uncertainties by incorporating inspection or monitoring data into prior probabilistic models (Moaveni et al., 2013). Machine learning tools such as Gaussian processes, neural networks and random forests provide surrogate models that approximate complex structural responses and support rapid reliability assessment (Zhang et al., 2023; Ma, 2024).

Several RC failure mechanisms have been investigated through probabilistic modelling. Chloride-induced corrosion has been analyzed using probabilistic diffusion models that account for random variability in concrete cover, diffusion coefficients and exposure conditions (Nogueira et al., 2012). Flexural and shear failures are evaluated by defining limit-state functions based on material strengths and load effects, followed by reliability analysis to compute safety indices (Lu et al., 1994).

Seismic fragility assessments quantify the probability of exceeding damage states for RC frames subjected to ground motions, accounting for uncertainty in hysteretic parameters, loading history and modelling assumptions (Blasi et al., 2023; Zeng et al., 2025). Multi-hazard studies combine deterioration and seismic effects, demonstrating that prior corrosion significantly increases collapse probability during earthquakes (Val et al., 2025).

Machine-learning-assisted surrogate models have been applied to blast and impact loading, providing fast estimators for failure probability where detailed nonlinear simulations would otherwise be too computationally expensive (Yang et al., 2025).

Model uncertainty is often larger than material or load variability when assessing existing RC structures (Achenbach, 2019). Calibration against experimental data, residual error quantification and Bayesian model updating can significantly reduce such uncertainties (Moaveni et al., 2013). Sensitivity analysis techniques such as Sobol indices or variance decomposition help identify key uncertain parameters driving failure (Melchers & Beck, 2018).

Translating probabilistic results into engineering decisions requires risk-based frameworks that incorporate target reliability indices, life-cycle cost optimization and prioritized maintenance planning (Ang & Tang, 2007). Time-dependent reliability supports optimal inspection scheduling and selection of retrofit strategies such as cathodic protection, section enlargement or fibre reinforced polymer (FRP) strengthening (Seghier et al., 2021).

Reliability-based design concepts were first formalized to address inconsistencies in safety margins produced by allowable stress and load factor design methods. Early work demonstrated that probabilistic calibration of design codes leads to more uniform safety levels across different structural components and loading scenarios (Cornell, 1969). This foundational work established the mathematical basis for defining reliability indices and failure probabilities in structural engineering applications.

Several studies have shown that reinforced concrete beam capacity predictions are strongly influenced by statistical assumptions regarding material strength distributions. Experimental investigations combined with probabilistic modelling indicate that assuming lognormal or normal distributions for concrete strength can significantly alter calculated failure probabilities, especially near ultimate limit states (Nowak & Szerszen, 2003). These findings emphasize the importance of consistent probabilistic characterization of material properties.

Spatial variability of concrete properties has also been shown to influence structural reliability. Random field modelling of concrete strength demonstrates that local weak zones can govern cracking patterns and ultimate failure, leading to lower reliability indices than those predicted using spatially uniform properties (Vanmarcke, 1983). Incorporating spatial randomness improves realism but increases computational demand.

Probabilistic assessment has been extensively applied to flexural reliability of RC beams under combined dead and live loading. Studies using Monte Carlo simulation and FORM show that live load variability dominates failure probability in short-span beams, while material variability becomes more critical for longer spans (Nowak & Collins, 2000). This distinction is important when prioritizing uncertainty sources in reliability models.

Time-dependent reliability of RC structures has gained attention due to ageing infrastructure worldwide. Probabilistic models integrating creep, shrinkage and reinforcement corrosion indicate that reliability indices decrease nonlinearly with time, particularly in aggressive environments (Enright & Frangopol, 1998). Such models provide a rational basis for life-cycle assessment and maintenance planning.

System reliability approaches extend member-level analysis by considering interaction between structural components. Research on RC frame systems shows that redundancy can significantly reduce overall system failure probability even when individual members exhibit relatively low reliability (Ditlevsen & Madsen, 1996). This highlights the limitations of component-based reliability assessment for complex structures.

Uncertainty in load modelling has been identified as a major contributor to failure probability. Statistical studies of live load data reveal that actual occupancy loads often differ significantly from code-specified nominal values, justifying the use of probabilistic live-load models in reliability analysis (Ellingwood & Tekie, 1999). These findings support the shift toward reliability-based load combinations.

Reliability analysis has also been applied to assess the safety of existing RC structures subjected to upgrading or change of use. Probabilistic evaluation allows engineers to quantify the impact of increased loads or material degradation and to compare retrofit options on a risk-informed basis (Stewart & Melchers, 1997).

Model uncertainty, arising from simplified resistance models and empirical design equations, has been shown to contribute significantly to overall uncertainty in RC reliability assessments. Calibration studies demonstrate that incorporating model bias and model uncertainty factors can substantially reduce unconservative reliability estimates (Nowak, 1995). This underscores the importance of including model uncertainty explicitly in probabilistic frameworks.

Recent research has emphasized the role of reliability analysis in sustainable and resilient infrastructure design. Reliability-based optimization enables reduction of material usage while maintaining acceptable safety levels, contributing to both economic and environmental sustainability (Frangopol & Soliman, 2016). This approach aligns structural safety assessment with modern performance- and resilience-based design philosophies.

In this study, Analytical mean-value and Morte Carlo Simulation probabilistic approaches will be used to explicitly model uncertainty sources such as variability in concrete compressive strength, steel yield stress, reinforcement placement errors and unpredictable load fluctuations to assess probability of failure in a simply supported reinforced concrete beam carrying uniformly distributed load.

II. MATERIALS AND METHOD

Description of structural element - The study considered the design and as-built of a simply supported reinforced concrete beam carrying uniformly distributed load subjected to flexural loading including the following properties

Table 1: Beam design and As-built properties

S/N	Design Dimension	Value
1	Length, L (m)	6
2	Width, b (mm)	250
3	Effective depth, d (mm)	500

S/N	As-built Dimension	Value
1	Length, L (m)	6
2	Width, b (mm)	230
3	Effective depth, d (mm)	450

Materials – The materials considered are those that directly influence the flexural performance of the beam, concrete, including reinforcing steel, and structural loads. Variability in material properties and loading conditions incorporated probabilistically to realistically represent uncertainties inherent in design and construction

Table 2: Beam design probabilistic properties

S/N	Random Variable	Mean (μ)	Std. Dev. (σ)	Probability Distribution
1	Length, L (m)	6	Deterministic	Deterministic
2	Width, b (mm)	250	Deterministic	Deterministic
3	Effective depth, d (mm)	500	Deterministic	Deterministic
4	Concrete strength, f'c (MPa)	30	4.5	Normal
5	Steel yield strength, f_y (MPa)	460	46	Normal
6	Steel area, A_s (mm^2)	1250	125	Normal
7	Uniform load, w (kN/m)	20	3	Normal

Table 3: Beam As-built probabilistic properties

S/N	Random Variable	Mean (μ)	Std. Dev. (σ)	Probability Distribution
1	Length, L (m)	6	Deterministic	Deterministic
2	Width, b (mm)	230	Deterministic	Deterministic
3	Effective depth, d (mm)	450	Deterministic	Deterministic
4	Concrete strength, f'c (MPa)	25	4	Normal
5	Steel yield strength, f_y (MPa)	420	50	Normal
6	Steel area, A_s (mm^2)	1000	150	Normal
7	Uniform load, w (kN/m)	30	5	Normal

Method – The probabilistic properties of the materials and loads form the basis for the analytical mean-value approach and the Monté Carlo simulation performed in this study. The flexural resistance and demand are evaluated using standard reinforced concrete beam theory, structural safety is assessed through a limit state function and probability of failure estimated from probability theory. Beam dimensions are deterministic (within construction tolerances ±5–10 mm), hence are significantly smaller and more controllable on site compared to materials properties and applied loads.

Reinforced concrete bean theory

Flexural resistance, $M_R = A_s f_y \, (d - \frac{a}{2})$ ……. equation (1)

Flexural resistance, $M_E = \frac{wL^2}{8}$ ……………equation (2)

Limit state function

Structural safety, $g = M_R - M_E$ …………...equation (3)

Probability theory

Probability of failure, $P_f \approx \frac{N_f}{N}$ ……………equation (4)

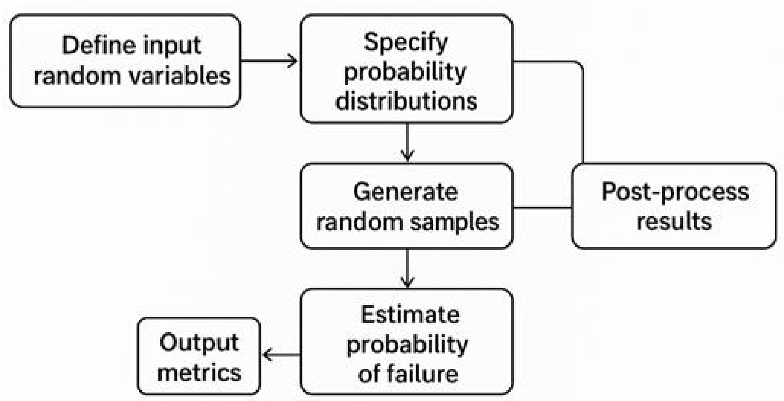

Fig 1: Standard Monte Carlo simulation flow chart
Source: Adapted from Rubinstein and Kroese (2016) and Investopedia

III. RESULTS AND DISCUSSION

Table 4: Beam design analytical mean-value approach

Sample	M_R (kNm)	M_E (kNm)	g	P_f	Status
30	319	90	229	0	Safe

In table 4, the analytical (mean-value) assessment indicates that the beam design is structurally safe, as the mean bending resistance M_R exceeds the mean bending demand M_E. A large safety margin (229kNm) was observed reflecting a conservative design.

Table 5: Beam As-built analytical mean value approach

Sample	M_R (kNm)	M_E (kNm)	g	P_f	Status
30	171	135	+	0	Safe

In table 5, the constructed beam shows a positive limit state value of 36kNm, although with a much smaller reserve capacity. This highlights that while the mean-value approach classifies the bean as safe, it does not capture the influence of variability in material properties and loading.

Table 6: Beam design Monte Carlo simulation

Simulation	M_R (kNm)	M_E (kNm)	g	P_f	Status
1	266.7	94.5	+172.2		Safe
2	247.3	81.0	+166.3		Safe
3	262.1	99.0	+163.1		Safe
4	225.8	108.0	+117.8		Safe
5	250.6	85.5	+165.1		Safe
6	258.4	103.5	+154.9		Safe
7	270.2	90.0	+180.2		Safe
8	230.5	112.5	+118.0	0	Safe
9	275.1	94.5	+180.6		Safe
10	240.6	99.0	+141.6		Safe
11	210.2	108.0	+102.2		Safe
12	290.8	85.5	+205.3		Safe
13	255.4	90.0	+165.4		Safe
14	220.5	117.0	+103.5		Safe
15	285.6	81.0	+20		Safe

Simulation	M_R (kNm)	M_E (kNm)	g	P_f	Status
16	266.7	94.5	+172.2		Safe
17	247.3	81.0	+166.3		Safe
18	262.1	99.0	+163.1		Safe
19	225.8	108.0	+117.8		Safe
20	250.6	85.5	+165.1		Safe
21	258.4	103.5	+154.9		Safe
22	270.2	90.0	+180.2		Safe
23	230.5	112.5	+118.0	0	Safe
24	275.1	94.5	+180.6		Safe
25	240.6	99.0	+141.6		Safe
26	210.2	108.0	+102.2		Safe
27	290.8	85.5	+205.3		Safe
28	255.4	90.0	+165.4		Safe
29	220.5	117.0	+103.5		Safe
30	285.6	81.0	+204.6		Safe

In table 6, Monte Carlo simulation for the beam design shows no failure cases across all thirty (30) simulations. In every realization, the bending resistance substantially exceeds the applied bending demand, resulting in consistently positive limit state values. This confirms that the design configuration possesses a high reliability level and adequate robustness against variability in loads and material properties.

Table 7: Beam As-built Monte Carlo simulation

Simulation	M_R (kNm)	M_E (kNm)	g	P_f	Status
1	144.0	151.6	+7.6		Safe
2	126.0	178.2	+52.2		Safe
3	153.0	147.1	-5.9		Safe
4	135.0	189.4	+54.4		Safe
5	139.0	165.2	+25.7		Safe
6	157.5	138.9	+18.6		Safe
7	130.5	198.6	+68.1		Safe
8	162.0	132.4	-29.6	0.3	Safe
9	148.5	149.8	+1.3		Safe
10	135.0	178.2	+43.2		Safe
11	153.0	142.0	-11.0		Safe
12	126.0	193.6	+67.6		Safe
13	135.0	165.2	+30.2		Safe
14	157.5	140.1	-17.4		Safe
15	121.5	198.6	+77.1		Safe

Simulation	M_R (kNm)	M_E (kNm)	g	P_f	Status
16	144.0	153.4	+9.4		Safe
17	148.5	147.8	-0.7		Safe
18	130.5	178.2	+47.7		Safe
19	139.5	165.2	+25.7		Safe
20	126.0	193.6	+67.6		Safe
21	157.5	136.7	-20.8		Safe
22	121.5	201.4	+79.9		Safe
23	144.0	153.4	+9.4	0.3	Safe
24	153.0	147.8	-5.2		Safe
25	135.0	178.2	+43.2		Safe
26	139.5	165.2	+25.7		Safe
27	130.5	188.0	+57.5		Safe
28	157.5	140.1	-17.4		Safe
29	135.0	178.2	+43.2		Safe
30	144.0	165.2	+21.2		Safe

In table 7, Monte Carlo simulation results for the constructed beam reveal a markedly different behaviour compared to the analytical approach. Out of 30 simulations, nine failure cases ($g<0$) are observed, corresponding to a probability of failure of approximately 0.30. These failures occur when relatively high load effects coincide with lower material strengths and reinforcement areas, demonstrating the sensitivity of the constructed beam to inherent uncertainties. This result indicates that although the beam is safe on a mean-value basis, it is probabilistically unreliable, with a significant likelihood of failure under unfavourable but realistic combinations of variables.

CONCLUSION AND RECOMMENDATIONS

1. The analytical (mean-value) method classified both the beam design and the constructed beam as safe; however, it did not capture the influence of variability in material properties and loading.
2. Monte Carlo simulation revealed that the constructed beam exhibits a significant probability of failure, despite being safe on a mean-value basis.
3. The beam design demonstrated high reliability, with no failure observed in all Monte Carlo simulations, indicating adequate safety margins.
4. The disparity between analytical and Monte Carlo results confirms that deterministic analysis may overestimate structural safety for as-built conditions.
5. Probabilistic analysis provides a more realistic representation of structural performance by accounting for uncertainties inherent in construction and loading.

Recommendations

- Probabilistic methods such as Monte Carlo simulation should complement conventional deterministic design checks, especially for existing or as-built structures.

- Greater emphasis should be placed on construction quality control to ensure that as-built properties align closely with design assumptions.
- Structures with reduced capacity or increased loading should undergo reliability assessment to identify potential failure risks.
- For critical structures, a larger number of Monte Carlo simulations should be employed to obtain more stable and representative probability estimates.
- Design codes and engineering practice should increasingly incorporate reliability-based approaches to enhance structural safety and decision-making..

REFERENCES

Achenbach, M. (2019). *Quantification of model uncertainties for reinforced-concrete recalculation results*. Engineering Structures, 181,43–56. https://doi.org/10.1016/j.engstruct.2018.10.020

Ang, A. H., & Tang, W. H. (2007). *Probability concepts in engineering planning and design* (Vol. 2). Wiley.

Beck, A. T., Souza Jr, M., & Nogueira, C. (2013). *Structural reliability analysis for engineering applications*. IBRACON.

Blasi, G., et al. (2023). *Fragility curves for reinforced concrete frames*. Engineering Structures. https://doi.org/10.1016/j.engstruct.2023.115348

Ellingwood, B., & Galambos, T. V. (1982). *Probability-based structural design*. Journal of Structural Engineering, 108(5), 959–977.

Ghannoum, M., et al. (2023). *Stochastic modelling of heterogeneity in concrete strength*. Buildings, 13(9), 2294. https://doi.org/10.3390/buildings13092294

Grubišić, M., et al. (2019). *Reliability analysis of reinforced concrete frames using MCS, FORM and SORM*. Buildings, 9(5), 119. https://doi.org/10.3390/buildings9050119

Lu, R., et al. (1994). *Reliability evaluation of reinforced concrete beams*. Structural Safety, 15, 21–34. https://doi.org/10.1016/0167-4730(94)90048-5

Ma, Y. (2024). *Bayesian probabilistic modelling for drift capacity of RC columns*. Probabilistic Engineering Mechanics. https://doi.org/10.1016/j.probengmech.2024.103507

Melchers, R. E., & Beck, A. T. (2018). *Structural reliability analysis and prediction* (3rd ed.). Wiley. https://doi.org/10.1002/9781119266101

Moaveni, B., et al. (2013). *Model updating using Bayesian inference*. Mechanical Systems and Signal Processing, 37, 123–139.

Neville, A. M. (2011). *Properties of concrete* (5th ed.). Pearson.

Nogueira, C. G., Leonel, E. D., & Coda, H. B. (2012). *Probabilistic failure modelling of RC structures subjected to chloride ingress*. International Journal of Advanced Structural Engineering, 4(1), 1–15. https://doi.org/10.1186/2008-6695-4-10

Seghier, M. E. A. B., et al. (2021). *Time-dependent reliability analysis of deteriorated RC structures*. Applied Sciences, 11(21), 10187. https://doi.org/10.3390/app112110187

Song, C., & Kawai, R. (2023). *Monte Carlo and variance reduction methods in structural reliability analysis: A review*. Reliability Engineering & System Safety, 231, 109078. https://doi.org/10.1016/j.ress.2022.109078

Val, D. V., et al. (2025). *Probabilistic modelling of deterioration in concrete structures*. Engineering Structures.

Yang, J., et al. (2025). *Machine-learning-based prediction of blast-induced damage in RC slabs*. Buildings, 15(2), 215. https://doi.org/10.3390/buildings15020215

Zeng, T., et al. (2025). *Collapse fragility analysis of RC frame structures*. Buildings, 15(1), 117. https://doi.org/10.3390/buildings15010117

Zhang, W., et al. (2023). *Probabilistic machine-learning framework for structural failure estimation*. Probabilistic Engineering Mechanics, 74, 103379. https://doi.org/10.1016/j.probengmech.2023.103379

Cornell, C. A. (1969). A probability-based structural code. *Journal of the American Concrete Institute*, 66(12), 974–985. https://doi.org/10.14359/7414

Ditlevsen, O., & Madsen, H. O. (1996). *Structural reliability methods*. John Wiley & Sons. https://doi.org/10.1002/9780470030480

Ellingwood, B. R., & Tekie, P. B. (1999). Wind load statistics for probability-based structural design. *Journal of Structural Engineering*, 125(4), 453–463.

https://doi.org/10.1061/(ASCE)0733-9445(1999)125:4(453)

Enright, M. P., & Frangopol, D. M. (1998). Maintenance planning for deteriorating concrete bridges. *Journal of Structural Engineering*, 124(12), 1407–1414. https://doi.org/10.1061/(ASCE)0733-9445(1998)124:12(1407)

Frangopol, D. M., & Soliman, M. (2016). Life-cycle of structural systems: Recent achievements and future directions. *Structure and Infrastructure Engineering*, 12(1), 1–20. https://doi.org/10.1080/15732479.2014.999794

Nowak, A. S. (1995). Calibration of LRFD bridge design code. *Journal of Structural Engineering*, 121(8), 1245–1251. https://doi.org/10.1061/(ASCE)0733-9445(1995)121:8(1245)

Nowak, A. S., & Collins, K. R. (2000). *Reliability of structures*. McGraw-Hill. https://doi.org/10.1036/0071349951

Nowak, A. S., & Szerszen, M. M. (2003). Calibration of design code for buildings (ACI 318): Part 1—Statistical models for resistance. *ACI Structural Journal*, 100(3), 377–382. https://doi.org/10.14359/12652

Stewart, M. G., & Melchers, R. E. (1997). *Probabilistic risk assessment of engineering systems*. Chapman & Hall. https://doi.org/10.1007/978-1-4757-2632-7

Vanmarcke, E. H. (1983). *Random fields: Analysis and synthesis*. MIT Press. https://doi.org/10.1007/978-1-4612-5194-6

