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Classical Methods and Henry Scheff's model on Compressive Strength of Concrete: a Comparative Approach

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ABSTRACT: *In the Building and construction processes, it is pertinent to know the compressive strength of concrete using casted cubes and obtain result of concrete cubes at 28th day. Usually three concrete cubes were casted to determine its strength. Using Henrys Scheff's polynomial approximation is quite cumbersome. This paper is an attempt to develop a simple mathematical model based on MATLAB R2007b code and compare it with that of Henry Scheff's Polynomial model. The model formulated compares favourably with the experiment data. It also satisfies the student's t and chi-square x^2 tests. The optimum value of concrete strength predicted by this model is 37.56N/mn^2 corresponding to a mix ratio of 1:1:2 of cement, sharp sand and chippings respectively at a water-cement ratio of 0.650.*

KEYWORDS: *Compressive strength, water cement ratio, mix proportion, curing ages. Henry Scheff's Polynomial, MathLab R2007b code.*

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I. INTRODUCTION

The versatile of concrete in construction practice for its availability, cheap rate, flexible of handling and giving shape of any desired form/ designing a concrete structure requires the concrete compressive strength to be used at 28th day strength.

Researchers are very keen to explore the concrete behavior and for this reason. Many studies are being carried out in this area Lambrakes (2013), Nwosu and I.M Akigwe (2020); Traditional modeling approaches are established based on empirical relation and experimental data which are improving day by day. Aleneme and Mbidike (2019) carried out the optimization of flexural strength of palm nut fibre concrete using Scheffes theory to predict the compressive strength of palm nut fibre concrete.

Nwachukwu (2022) development and used Scheffes third degree polynonid model to optimize the compressive strength of polypropylene Fibre reinforce concrete Sule (2018) investigated the structural characteristics of Chikoko mud-cement using Scheffes theory.

Anya (2014) developed a model for predicting the structural characteristics of sand quary dust blocks for a 4-component mixture.

Ndububa (2018) carried a mixture design in fibre cement production by developing a mathematical model using Scheffes that could predict mix proportions).

Sule S. (2018) determining the structure characteristics of chikoko mud-cement using Scheffes and Osadebe's techniques. Futospec futo.edu.ng.

II. MATERIALS AND METHODS

A. MATERIALS

Materials required for the experiment include sample of unwashed chipping from Onitsha River. The sharp sand collected was stored in cement bags in a closed room. Cement from Dangote.

B. METHOD

By means of a weighing balance; water, cement, fine aggregate and coarse aggregate were weighed our respectively in the proportions shown in Table I right such that the materials weighed will serve for three cubes. Three cubes were cast from each of the proportions making 60 cubes in all the fresh concrete was filled into the 150mm x 150mm steel mould and tamped for 20 times. The concrete was allows to harden for 24hours. The cubes were removed and put in a water tank for 28 days. At the end of 28 days water cured, the cubes were crushed in the universal crushing machine.

The results are averages from the test points were tabulate in column 7 to 10 of Table 2. Extra ten test points were provided for validation of the model.

a. Model development

Simplex lattice design proposed by Henry Scheffe (1958) was used to formulate a mathematical model which relates compressive strength of concrete and its component rations of cement, sharp sand, chippings and water contents.

The reduced second degree polynomial for quaternary system is derived as follows.

$Y = bo + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_{12}X_1X_2 + b_{13}X_1X_3 +$

$b_{14}X_1X_4 + b_{22}X_2X_3 + b_{24}X_2X_4 + b_{34}X_3X_4 + b_{11}X_1^2X_4 + b_{11}X_1^2 + X_2^2 +$

$b_{33}X_3^2 + b_{44}X_4^2 +$ ………………………………………..…………………………….... 1

Since $X_1 + X_2 + X_3 + X_4 =$ ……………………………………………..…………. 2

Multiplying equation (10) by bo we have

$boX_1 + boX_2 + boX_3 + boX_4 = bo$ ……………………………………….……..… 3

Multiplying equation (9) by $X_1X_2X_3$ and X_4 in

Succession gives

$X_1^2 = X_1 - X_1X_2 - X_1X_3 - X_1X_4$

$X_2^2 = X_2 - X_1X_2 - X_2X_3 - X_2X_4$ ……………………………………………...……. 4

$X_3^2 = X_3 - X_1X_3 - X_2X_3 - X_3X_4$

$X_4^2 = X_4 - X_1X_4 - X_2X_3 - X_4X_4$

Substituting equation (11) and equation (12) into equation (9) and transforming it gives.

$\overset{n}{Y} = (bo + b_1 + b_{11})\, X_1 + (bo + b_2 + b_{22})\, X_2 + (bo + b_3 + b_{33})\, X_3 +$

$(bo + b_4 + b_{44})\, X_4 + (b_{12} + b_{11} + b_{22})\, X_1X_2 + (b_{13} - b_{11}) -$

$b_{33})X_1X_3 + b_{14} + b_{11}) - b_{44})X_1X_4 + (b_{23} - b_{23} - b_{23})\, X_2X_3$

$+ (b_{24} + b_{22} - b_{44})X_2X_4 + (b_{34} - b_{33} - b_{44})\, X_3X_4$ ……………………….………… 5

Denoting

Bi = bo + bi + bii; Bij = bij – bii – bij …………………….…………………….…..………… 6

The reduced second-degree polynomial in four variables is then arrived at as:

$$\overset{n}{Y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4$$

$$+ \beta_{23}X_2X_3 + \beta_{23}X_2X_4 + \beta_{34}X_3X_4 \quad \ldots\ldots 7$$

The solution of equation (6) as given by Henry Scheffe for the coefficients of the polynomial is Bi = Yi and bij = 4Yij – 2Yi – 2Yj ……………………………..…………………….………………………… 8

Where $\beta i = \beta_1, \beta_2, \beta_3 \ldots\ldots\ldots \beta_4$

$\beta ij = \beta_{12}, \beta_{13}, \beta_{14} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \beta_{34}$

Yi and Yij = response (property) that is, experimental results.

Equation (7) is the governing equation.

Simplex Lattice

For a 4-component mixture, we have a tetrahedron simplex-lattice as shown in Fig. 1.

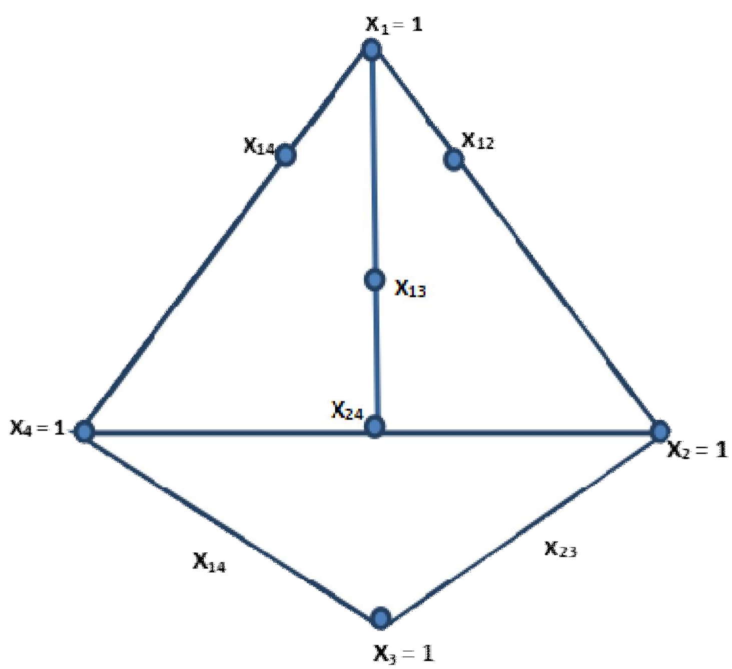

Fig.1: Factor Notation for A (4,2) – Lattice Polynomial

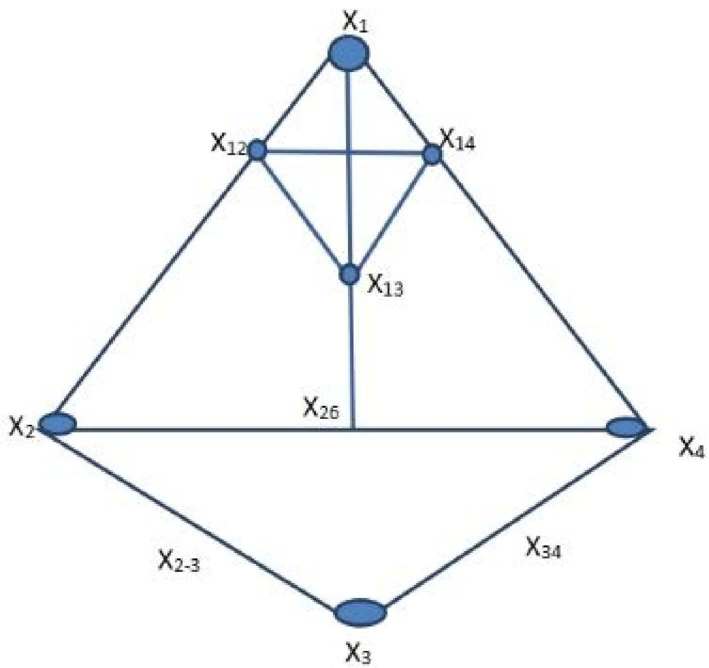

Fig.2: Factor notation for a (4,2) – lattice

Table 1: Matrix table for Scheffe`s (4,2) – Lattice Polynomial

Pseudo - Components					Response	Real Components			
S/N	X_1	X_2	X_3	X_4		Z_1	Z_2	Z_3	Z_4
1	1	0	0	0	Y_1	0.6	1.0	1.5	4
2	0	1	0	0	Y_2	0.5	1.0	1.0	1½
3	0	0	1	0	Y_3	0.55	1.0	1½	3.0
4	0	O	0	1	Y_4	0.555	1.0	2½	4.0
5	½	½	0	0	Y_{12}	0.55	1.0	1.25	2.75
6	½	0	½	0	Y_{13}	0.575	1.0	1.5	3.5
7	½	0	0	½	Y_{14}	0.578	1.0	2.0	4.0
8	0	½	½	0	Y_{23}	0.525	1.0	1.25	2.25
9	0	½	0	½	Y_{24}	0.528	1.0	1.75	2.75
10	0	0	½	½	Y_{34}	0.553	1.0	2.0	3.5

For the fact that concrete mixtures have its sum of proportions above unity a congruent simplex is necessary such that the mix proportions at the vertices show the range of w/c ratio, cement, fine aggregate and coarse aggregate ratios, respectively, the required polynomial model will cover or predict (see Fig.2)

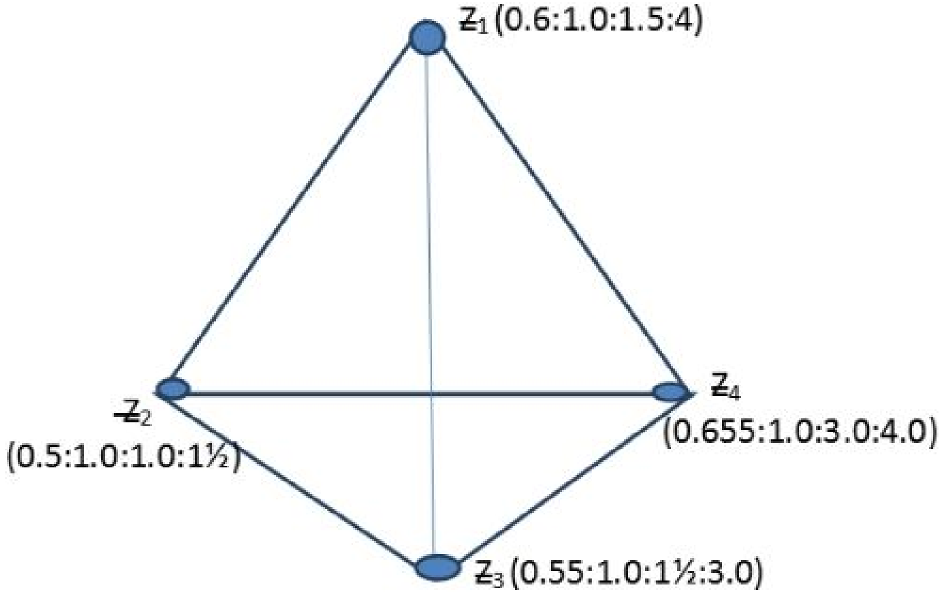

Fig 3: Real Component Simplex (only vertices are shown)

The former simplex, Fig.1 is called Pseudo- component simplex and later, Fig.2, component simplex. From the later (real components) a Z-matrix is formed whose transpose becomes the conversion factor from pseudo to real component; i.e. from Fig.2.

$$Z = \begin{pmatrix} 0.6 & 1.0 & 1.5 & 4.0 \\ 0.5 & 1.0 & 1.0 & 1\tfrac{1}{2} \\ 0.55 & 1.0 & 1\tfrac{1}{2} & 3.0 \\ 0.55 & 1.0 & 2\tfrac{1}{2} & 4.0 \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 9$$

$$\text{and } Z^T \begin{pmatrix} 0.6 & 0.5 & 0.55 & 0.555 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 1.5 & 1.0 & 1\tfrac{1}{2} & 2\tfrac{1}{2} \\ 4.0 & 1\tfrac{1}{2} & 3.0 & 4.0 \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 10$$

To demonstrate the use of eq(11) in Table 1, the 5^{th} row in the row in the real component side is obtained by multiplying {z}t matrix by the corresponding row in the pseudo- component side of the Table 1.0, i.e.

$$\begin{pmatrix} 0.6 & 0.5 & 0.55 & 0.555 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 1.5 & 1.0 & 1½ & 2½ \\ 4.0 & 1½ & 3.0 & 4.0 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.55 \\ 1.0 \\ 1.25 \\ 2.75 \end{pmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad 9$$

In this way all component rows in the real component side are obtained producing a congruent table and simplex.

Table 2: Actual (Zi) and Pseudo (Xi) Components for the Ten Design Experimental Points of (4,2) Lattice laboratory result

N	X1	X2	X3	X4	Response	Z1	Z2	Z3	Z4
1	1	0	0	0	Y1	0.65	1	1	2
2	0	1	0	0	Y2	0.75	1	2	4
3	0	0	1	0	Y3	0.10	1	3	6
4	0	0	0	1	Y4	1.14	1	3	8
5	0.5	0.5	0	0	Y12	0.70	1	1.5	3
6	0.5	0	0.5	0	Y13	0.875	1	2	4
7	0.5	0	0	0.5	Y14	0.895	1	2	5
8	0	0.5	0.5	0	Y23	0.925	1	2.5	5
9	0	0.5	0	0.5	Y24	0.945	1	2.5	6
10	0	0	0.5	0.5	Y34	1.12	1	3	7

Table 3: Actual (Zi) and Pseudo (Xj) Components for Ten Control Test Points of (4,2) Lattice

N	X1	X2	X3	X4	Cexp	Z1	Z2	Z3	Z4
1	0.75	0.25	0	0	C1	0.675	1	1.25	2.5
2	0	0.75	0	0.25	C2	0.848	1	2.25	5
3	0.25	0	0.75	0	C3	0.988	1	2.5	5
4	0	0.25	0.75	0	C4	1.013	1	2.75	5.5
5	0.5	0.25	0.25	0	C5	0.788	1	1.75	3.5
6	0.6	0	0.25	0.25	C6	0.885	1	2	4.5
7	0.25	0	0.50	0.25	C7	0.998	1	2.5	5.5
8	0	0.25	0.50	0.25	C8	1.023	1	2.75	6
9	0.25	0	0.25	0.5	C9	0.998	1	2.5	6
10	0.25	0.25	0.25	0.25	C10	0.910	1	2.25	5

III. RESULTS AND DISCUSSION

A. THE REGRESSIVE EQUATION

Reference to Equation 7 and Table 3 The coefficients of the second degree polynomial equation are determined as follows:

$\beta_1 = Y_1 = 37.56$

$\beta_2 = Y_2 = 24.12$

$\beta_3 = Y_3 = 24.00$

$\beta_4 = Y_4 = 14.89$

$\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2 = 4(11.34)-2(37.56)-2(24.12) = -78$

$\beta_{13} = 4Y_{13} - 2Y_1 - 2Y_3 = 4(34)-2(37.56)-2(24) = 12.88$

$\beta_{14} = 4Y_{14} - 2Y_1 - 2Y_4 = 4(15)-2(37.56)-2(14.89) = -44.9$

$\beta_{23} = 4Y_{23} - 2Y_2 - 2Y_3 = 4(23.78)-2(24.12)-2(24) = 1.12$

$\beta_{24} = 4Y_{24} - 2Y_2 - 2Y_4 = 4(18.89)-2(24.12)-2(14.89) = -2.46$

$\beta_{34} = 4Y_{34} - 2Y_3 - 2Y_4 = 4(15.11)-2(24)-2(14.89) = -17.34$

Then, from equation 7

$$Y = 37.56\, x_1 + 24.12\, x_2 + 24.0\, x_3 + 14.89\, x_4 - 78\, x_1 x_2 + 12.88\, x_1 x_3 - 44.9\, x_1 x_4 - 1.12\, x_2 x_3 - 2.46\, x_2 x_4 - 17.34\, x_3 x_4 \quad \ldots\ldots\, 11$$

Equation 11 is the regression equation for the compressive strength of concrete as obtained in this study.

Table 1: Result of Individual Coefficient contribution for estimating CS

Predictor	Coefficient	Standard Error Coefficient	T-value	P
X	42.490	3.009	14.12	0.000
y	-0.972	3.804	-0.26	0.801
2	-4.141	1.468	-2.82	0.012

Standard Error = 3.08004 R-Square = 83.8% R-Square(adjusted) = 81.9%

The result obtained in table 1 found a strongly positive coefficient of determination {R-square =83.8%) which implies that the independent variables (mix ratio) were able to explain about 83.8% of total variation in compressive strength (CS). Also, the coefficient for X was found to be positive while Y and 2 were found to be negative. This result indicates that the variable X has a positive relationship with the CS while Y and Z has negative or inverse relationship with CS. It was found that only variable Z has a significant impact on CS with, a p-value =0.012 which is less than 5% critical value. The appropriate model for estimating compressive strength was found to be equation (4.1).

Table 4: Analysis of Variance (ANOVA) for estimating CS

Source	Degree of Freedom	Sum of Squares	Mean Square	F-value	P-value
Regression	2	832.15	416.07	43.86	0.000
Residual Error	17	161.27	9.49		
Total	993.42				

The result of the ANOVA analysis presented in table 2 found F-value of 43.86 and p-value of 0.00 which is less than 5% critical value. This result indicates that the independent variables significantly impact on the model and useful in estimating the values of CS.

Table 5: Result of Actual values of CS and Fitted/Estimated (FITSCS) values of CS

S/NO	X	Y	Z	CS	FITSCS
1	1	1	2	37.555	33.2347
2	1	2	4	24.22	23.9796
3	1	2	4	24	23.9796
4	1	3	6	14.89	14.7246
5	1	3	8	11.335	6.4417
6	1	1.5	3	34	28.6072
7	1	2	5	15	19.8382
8	1	2.5	5	23.775	19.3521
9	1	2.5	5	18.89	19.3521
10	1	2.5	6	15.11	15.2106
11	1	2.5	6	13.12	15.2106
12	1	3	7	9.11	10.5831
13	1	1.25	2.5	26.11	30.9209
14	1	2.25	5	17,335	19.5951
15	1	2.75	5.5	16	17.0383
16	1	1.75	3.5	24.22	26.2934
17	1	2	4.5	22.445	21.9089
18	1	2,5	5.5	18.885	17.2814
19	1	2.75	6	14.115	14.9676
20	1	2.25	5	18	19.5951

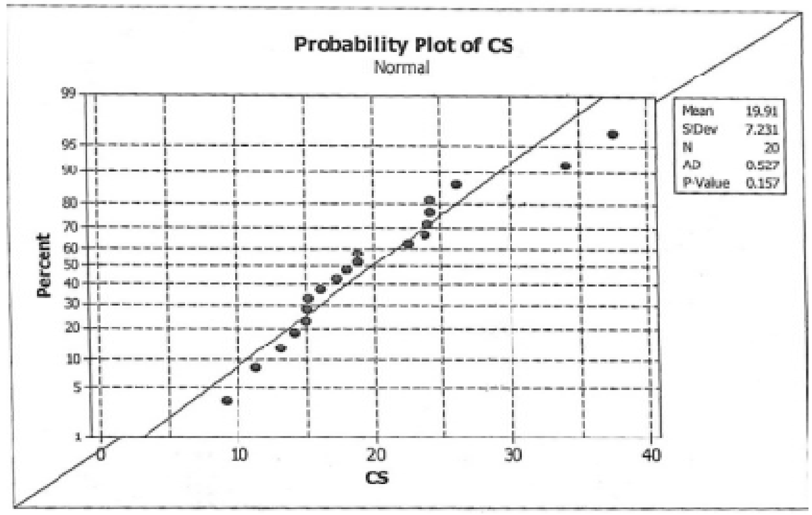

Fig. 4: Graph showing the result of Anderson-Darling Normality Test

The result found in figure 2 found a Anderson-Darling (AD) value of 0.527 and p-value of 0.157 which implies that the distribution of CS is normally distributed since p-value =0.527 is greater than significant value of 0.05 assuming 95% confidence level. -.' ;

MATLAB R2007b code for running regression Analysis

A=[l l 2; 1 2 4; 1 2 4; 1 3 6; 1 3 8; 11.5 3; 12 5; 1 2.5 5; 112.5 5; 12.5 6; 12,5 6; 13 7; 11.25 2.5; 1 2.25 5; 1 2.75 5,5; 11.75 3.5; 1 2 4.5; 1 2.5 5.5; 1 2.75 6; 12.25 5].

B = [37.555; 24.22; 24; 14.89; 11.335; 34.0; 15; 23.775; 18.89; 15.11; 13.12; 9.11; 26,11; 17.335; 16.00; 24.22; 22.445; 18.885; 14.115; 18.00]

>> R = regress(B,A)

CS = 42.5X - 0.97 Y - 4.14 Z

Table 6: Comparison of Results

Concrete Mix	Henry Scheffs N/mn^2	Mathlab on Regression Equation
1:1:2	37.56	33.25
1:2:4	24.12	24.00
1:2:4.5	14.89	21.00

From the comparison in table 6 above, it is evident that Henry Scheffs and MathLab on Regression Equation for compressive strength concrete compares favourably. It is important to adopt MathLab analysis because it offers the following advantages of classical method of Henry Scheffes model:

1. Rigorous equation and matrices are eliminated.
2. No need for strength of concrete response and central points.

3. Classical method (MATLAB, 2007b code) offers the regression equation and graph strength of concrete test result and solve all the simultaneous equations represented in all matrices. Once the mix ratio for concrete are prescribed, the equation generates the strength required.

IV. CONCLUSION

m this work, the performance of alkyd resin coated recycled aggregate were determined. Based on the experimental study conducted, the following conclusions were made:

1. The results showed that increase in concentration of coating improved the compressive strength of concrete.
2. Noticeable strength improvements were observed for increased in coated recycled aggregate concrete. The workability of fresh concrete produced from alkyd resin coated recycled coarse aggregate increased when compared with uncoated recycled coarse aggregate. This was in conformity with studies by Ryou and Lee, (2014).
3. The increased in concentration of alkyd resin coating of the recycled coarse aggregate reduces water absorption and improves workability.

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Graph showing the result of Anderson-Darling Normality Test for probability plot of CS.

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